Pre-Calculus **HW#5.4-3** **Name:** Key

No Notes (sections 5.2-5.3)

For each of the following, find the <u>exact</u> value of the expression.

1. $\sin\frac{5\pi}{12} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

$\sin\left(\frac{2\pi}{12}+\frac{3\pi}{12}\right)$

$\sin\frac{\pi}{6}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{6}$

$\frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$

$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

2. $\cos 285° = \boxed{\frac{-\sqrt{2}+\sqrt{6}}{4}}$

$\cos(240° + 45°)$

$\cos 240° \cos 45° - \sin 240° \sin 45°$

$-\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$-\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

3. $\sin\frac{7\pi}{12}\cos\frac{\pi}{12} - \sin\frac{\pi}{12}\cos\frac{7\pi}{12} = \boxed{1}$

$\sin\left(\frac{7\pi}{12} - \frac{\pi}{12}\right)$

$\sin\frac{6\pi}{12}$

$\sin\frac{\pi}{2}$

1

4. $2\sin 22.5° \cos 22.5°$

$\sin(2\cdot 22.5°)$

$\sin 45°$

$\boxed{\frac{\sqrt{2}}{2}}$

5. $\cos^2 15° - \sin^2 15°$

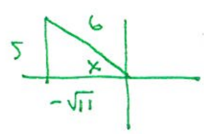
$\cos(2\cdot 15°)$

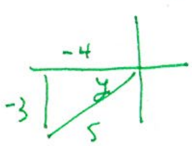
$\cos 30°$

$\boxed{\frac{\sqrt{3}}{2}}$

Use the given information to find the exact value of the expression.

6. If $\sin x = \frac{5}{6}$ and x lies in QII, $\tan y = \frac{3}{4}$ and y lies in QIII, find $\sin(x+y)$.





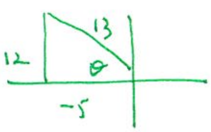
$\sin(x+y) = \sin x \cos y + \sin y \cos x$

$= \frac{5}{6}\cdot\frac{-4}{5} + \frac{-3}{5}\cdot\frac{-\sqrt{11}}{6}$

$= \frac{-20}{30} + \frac{3\sqrt{11}}{30}$

$= \boxed{\frac{-20+3\sqrt{11}}{30}}$ or $-\frac{2}{3} + \frac{\sqrt{11}}{10}$

7. If $\sin\theta = \frac{12}{13}$ and θ lies in QII, find $\cos 2\theta$ and $\sin 2\theta$.



1 $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \frac{25}{169} - \frac{144}{169}$

$= \boxed{\frac{-119}{169}}$

2 $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cdot\frac{12}{13}\cdot\frac{-5}{13}$

$= \boxed{\frac{-120}{169}}$

Verify the following.

8. $\dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

$\dfrac{\sin\alpha\cos\beta + \sin\beta\cos\alpha}{\cos\alpha\cos\beta}$

$\dfrac{\sin\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}} + \dfrac{\sin\beta\cancel{\cos\alpha}}{\cancel{\cos\alpha}\cos\beta}$

$\tan\alpha + \tan\beta$

9. $\cot x = \dfrac{1+\cos 2x}{\sin 2x}$

$\dfrac{1+\cos^2 x - \sin^2 x}{2\sin x\cos x}$

$\dfrac{1 + 1 - \sin^2 x - \sin^2 x}{2\sin x \cos x}$

$\dfrac{2 - 2\sin^2 x}{2\sin x\cos x}$

$\dfrac{\cancel{2}(1-\sin^2 x)}{\cancel{2}\sin x\cos x}$

$\dfrac{\cos^{\cancel{2}} x}{\sin x\cancel{\cos x}}$

$\cot x$

Notes OK (sections 5.3-5.4)

10. Find the exact value of the expression using a half-angle formula: $\sin 105°$

$\sin\dfrac{210°}{2} = +\sqrt{\dfrac{1-\cos 210°}{2}}$

$= \sqrt{\dfrac{1+\frac{\sqrt{3}}{2}}{2}}$

$\sqrt{\dfrac{\frac{2+\sqrt{3}}{2}}{\frac{2}{1}}} = \sqrt{\dfrac{2+\sqrt{3}}{4}} = \boxed{\dfrac{\sqrt{2+\sqrt{3}}}{2}}$

11. Find the exact value of $\cos 75° - \cos 15°$

$-2\sin\left(\dfrac{75°+15°}{2}\right)\sin\left(\dfrac{75°-15°}{2}\right)$

$-2\sin 45°\sin 30°$

$-2\cdot\dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2} \longrightarrow \boxed{-\dfrac{\sqrt{2}}{2}}$

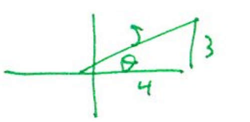
12. Find the exact value of $\sin\dfrac{\pi}{12} - \sin\dfrac{5\pi}{12}$

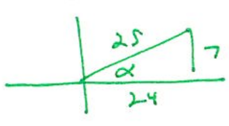
$2\sin\left(\dfrac{\frac{\pi}{12}-\frac{5\pi}{12}}{2}\right)\cos\left(\dfrac{\frac{\pi}{12}+\frac{5\pi}{12}}{2}\right)$

$2\sin\left(-\dfrac{\pi}{6}\right)\cos\dfrac{\pi}{4}$

$2\cdot -\dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2} \longrightarrow \boxed{-\dfrac{\sqrt{2}}{2}}$

13. Given: $\sin\theta = \dfrac{3}{5}$, $\cos\alpha = \dfrac{24}{25}$ and both θ and α lie in QI, find $\cos\dfrac{\theta}{2}$ and $\sin\dfrac{\alpha}{2}$.





① $\cos\dfrac{\theta}{2} = +\sqrt{\dfrac{1+\cos\theta}{2}}$

$= \sqrt{\dfrac{1+\frac{4}{5}}{2}}$

$= \sqrt{\dfrac{9}{10}}$

$= \dfrac{3}{\sqrt{10}}\cdot\dfrac{\sqrt{10}}{\sqrt{10}} = \boxed{\dfrac{3\sqrt{10}}{10}}$

② $\sin\dfrac{\alpha}{2} = +\sqrt{\dfrac{1-\cos\alpha}{2}}$

$= \sqrt{\dfrac{1-\frac{24}{25}}{2}}$

$= \sqrt{\dfrac{1}{50}}$

$= \dfrac{1}{5\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = \boxed{\dfrac{\sqrt{2}}{10}}$

Verify the following identity.

14. $\dfrac{\sin 3x - \sin x}{\cos 3x - \cos x} = -\cot 2x$

$\dfrac{2\sin\left(\frac{3x-x}{2}\right)\cos\left(\frac{3x+x}{2}\right)}{-2\sin\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)}$

$\dfrac{\cancel{2}\cancel{\sin x}\cos 2x}{-\cancel{2}\sin 2x\cancel{\sin x}}$

$-\dfrac{\cos 2x}{\sin 2x}$

$-\cot 2x$