

Solve the following equations over the interval $[0, 2\pi)$.

1. $(\cos x + \sqrt{3})(2\sin x + 1) = 0$

$$\cos x = -\sqrt{3} \quad \sin x = -\frac{1}{2}$$

\emptyset

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

2. $(\tan x - 1)(\cos x + 1) = 0$

$$\tan x = 1 \quad \cos x = -1$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}, \pi$$

3. $\sin x + 2\sin x \cos x = 0$

$$\sin x (1 + 2\cos x) = 0$$

$$\sin x = 0 \quad \cos x = -\frac{1}{2}$$

$$x = 0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$$

4. $\tan^2 x \cos x = \tan^2 x$

$$\tan^2 x \cos x - \tan^2 x = 0$$

$$\tan^2 x (\cos x - 1) = 0$$

$$\tan x = 0 \quad \cos x = 1$$

$$x = 0, \pi$$

5. $2\sin^2 x - \sin x - 1 = 0$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{2} \quad \sin x = 1$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$$

6. $\cos^2 x + 2\cos x - 3 = 0$

$$(\cos x + 3)(\cos x - 1) = 0$$

$$\cos x = -3 \quad \cos x = 1$$

$$\emptyset \quad x = 0$$

7. $2\cos^2 x + \sin x - 1 = 0$

$$2(1 - \sin^2 x) + \sin x - 1 = 0$$

$$+ 2\sin^2 x + \sin x + 1 = 0$$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{2} \quad \sin x = 1$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$$

8. $\sin^2 x - 2\cos x - 2 = 0$

$$1 - \cos^2 x - 2\cos x - 2 = 0$$

$$+\cos^2 x + 2\cos x + 1 = 0$$

$$(\cos x + 1)(\cos x + 1) = 0$$

$$\cos x = -1$$

$$x = \pi$$

9. $\sin 2x = \cos x$

$$2\sin x \cos x - \cos x = 0$$

$$\cos x (2\sin x - 1) = 0$$

$$\cos x = 0 \quad \sin x = \frac{1}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$

10. $\cos 2x + 5\cos x + 3 = 0$

$$2\cos^2 x - 1 + 5\cos x + 3 = 0$$

$$2\cos^2 x + 5\cos x + 2 = 0$$

$$(2\cos x + 1)(\cos x + 2) = 0$$

$$\cos x = -\frac{1}{2} \quad \cos x = -2$$

\emptyset

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

11. $\sin(x + \frac{\pi}{4}) + \sin(x - \frac{\pi}{4}) = 1$

$$\sin x \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos x + \sin x \cos \frac{\pi}{4} - \sin \frac{\pi}{4} \cos x = 1$$

$$2\sin x \cos \frac{\pi}{4} = 1$$

$$2 \cdot \frac{\sqrt{2}}{2} \sin x = 1$$

$$\sin x = \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}$$