Pre-Calculus **HW#5. 5-4** **Name:** Key

List all solutions over the interval $[0,2\pi)$.

1. $\csc x+2=0$

$\csc x = -2$

$\sin x = -\frac{1}{2}$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

2. $3\tan x-3=0$

$\tan x = 1$

$\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

3. $\sin 2x-\sin x=0$

$2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0$ or $\cos x = \frac{1}{2}$

$\boxed{x = 0, \pi \quad x = \frac{\pi}{3}, \frac{5\pi}{3}}$

4. $\sin(x+\frac{\pi}{3})+\sin(x-\frac{\pi}{3})=1$

$\sin x\cos\frac{\pi}{3} + \cancel{\sin\frac{\pi}{3}\cos x} + \sin x\cos\frac{\pi}{3} - \cancel{\sin\frac{\pi}{3}\cos x} = 1$

$2\sin x\cos\frac{\pi}{3} = 1$

$2\cdot\frac{1}{2}\sin x = 1$

$\sin x = 1$

$\boxed{x = \frac{\pi}{2}}$

5. $6\cos^2 x-5\sin x=2$

$6(1-\sin^2 x) - 5\sin x - 2 = 0$

$6 - 6\sin^2 x - 5\sin x - 2 = 0$

$-6\sin^2 x - 5\sin x + 4 = 0$

$6\sin^2 x + 5\sin x - 4 = 0$

$(2\sin x - 1)(3\sin x + 4) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -\frac{4}{3}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$ $\downarrow$ no solution

6. $2\cos x\csc x-4\cos x-\csc x=-2$

$2\cos x\csc x - 4\cos x - \csc x + 2 = 0$

$2\cos x(\csc x - 2) - 1(\csc x - 2) = 0$

$(\csc x - 2)(2\cos x - 1) = 0$

$\csc x = 2$ or $\cos x = \frac{1}{2}$

$\sin x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6} \quad x = \frac{\pi}{3}, \frac{5\pi}{3}}$

7. Use a half-angle formula to solve: $2-\sin^2 x=2\cos^2\frac{x}{2}$

$2 - \sin^2 x = 2\left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2$

$2 - \sin^2 x = \cancel{2}\left(\frac{1+\cos x}{\cancel{2}}\right)$

$2 - \sin^2 x = 1 + \cos x$

$2 - (1 - \cos^2 x) - 1 - \cos x = 0$

$\to 2 - 1 + \cos^2 x - 1 - \cos x = 0$

$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

$\cos x = 0$ or $\cos x = 1$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2} \quad x = 0}$

Find all solutions (in general form).

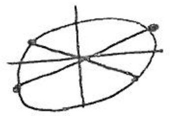
8. $\tan x = \dfrac{1}{3\tan x}$

$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}}$

$\tan x = \pm\frac{\sqrt{3}}{3}$



$\boxed{x = \frac{\pi}{6} + \pi n, \ \frac{5\pi}{6} + \pi n}$

9. $\sec^2(2x) = \sec(2x) + 2$

$\sec^2 2x - \sec 2x - 2 = 0$

$(\sec 2x - 2)(\sec 2x + 1) = 0$

$\sec 2x = 2$ or $\sec 2x = -1$

$\cos 2x = \frac{1}{2}$ $\quad \cos 2x = -1$

$2x = \frac{\pi}{3} + 2\pi n, \ \frac{5\pi}{3} + 2\pi n$ $\quad 2x = \pi + 2\pi n$

$\boxed{x = \frac{\pi}{6} + \pi n, \ \frac{5\pi}{6} + \pi n, \ \frac{\pi}{2} + \pi n}$

10. $\sin^2(3x) - 2\sin(3x) + 1 = 0$

$(\sin 3x - 1)(\sin 3x - 1) = 0$

$\sin 3x = 1$

$3x = \frac{\pi}{2} + 2\pi n$

$\boxed{x = \frac{\pi}{6} + \frac{2\pi}{3} n}$

11. $\tan(3t) = -\sqrt{3}$

$3t = \frac{2\pi}{3} + \pi n$ $\quad 3t = \frac{5\pi}{3} + \pi n$

$\boxed{t = \frac{2\pi}{9} + \frac{\pi}{3} n, \ \frac{5\pi}{9} + \frac{\pi}{3} n}$

12. $\sec^2 x - 3\tan x = 5$

$\tan^2 x + 1 - 3\tan x - 5 = 0$

$\tan^2 x - 3\tan x - 4 = 0$

$(\tan x - 4)(\tan x + 1) = 0$

$\tan x = 4$ $\quad \tan x = -1$

$\downarrow$

calculator

$\boxed{x = \frac{3\pi}{4} + \pi n}$

$\boxed{x = 1.33 + \pi n \text{ or } 4.47 + \pi n}$

13. $\sin 2x\cos x + \cos 2x \sin x = \dfrac{\sqrt{2}}{2}$

$\sin(2x + x) = \frac{\sqrt{2}}{2}$

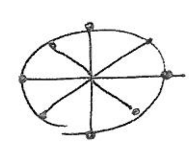
$\sin 3x = \frac{\sqrt{2}}{2}$

$3x = \frac{\pi}{4} + 2\pi n$ $\quad 3x = \frac{3\pi}{4} + 2\pi n$

$\boxed{x = \frac{\pi}{12} + \frac{2\pi}{3} n, \ \frac{\pi}{4} + \frac{2\pi}{3} n}$

14. Use a sum-to-product formula to solve: $\sin 5x + \sin 3x = 0$

$2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right) = 0$



$2\sin 4x \cos x = 0$

$\sin 4x = 0$ or $\cos x = 0$

$4x = 0 + 2\pi n, \ \pi + 2\pi n$ $\quad x = \frac{\pi}{2} + 2\pi n, \ \frac{3\pi}{2} + 2\pi n$

$x = 0 + \frac{\pi}{2} n, \ \frac{\pi}{4} + \frac{\pi}{2} n, \ \frac{\pi}{2} + 2\pi n, \ \frac{3\pi}{2} + 2\pi n$

$\boxed{x = 0 + \frac{\pi}{2} n, \ \frac{\pi}{4} + \frac{\pi}{2} n}$ or $\boxed{x = 0 + \frac{\pi}{4} n}$:)