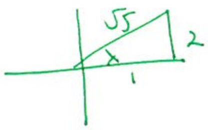
**Pre-Calculus** **Review Ch. 5, Part 2 (Notes ok)** **Name:** Key

**1.** If $\sin x = \dfrac{2}{\sqrt{5}}$ and x is in quadrant I, find $\sin 2x$ and $\cos 2x$.




$$\sin 2x = 2\sin x\cos x = 2\cdot\frac{2}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}} = \boxed{\frac{4}{5}}$$

$$\cos 2x = \cos^2 x - \sin^2 x = \frac{1}{5} - \frac{4}{5} = \boxed{\frac{-3}{5}}$$

**2.** Evaluate, using sum/difference formulas: $\sin\dfrac{\pi}{12}$.

$$\sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

**3.** Evaluate, using half-angle formulas: $\sin\dfrac{3\pi}{8}$

$$\sin\frac{3\pi}{8} = \sin\left(\frac{\frac{3\pi}{4}}{2}\right) = +\sqrt{\frac{1-\cos\frac{3\pi}{4}}{2}} = \sqrt{\frac{1 - -\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{\frac{2}{1}}} = \sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}}$$

$$\sqrt{\frac{2+\sqrt{2}}{4}} = \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$$

**4.** Use a product-to-sum formula to simplify: $\sin\dfrac{\pi}{12} - \sin\dfrac{5\pi}{12}$

$$2\sin\left(\frac{\frac{\pi}{12}-\frac{5\pi}{12}}{2}\right)\cos\left(\frac{\frac{\pi}{12}+\frac{5\pi}{12}}{2}\right)$$

$$2\sin\left(-\frac{4\pi}{24}\right)\cos\left(\frac{6\pi}{24}\right)$$

$$2\sin\left(-\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) = 2\cdot\frac{-1}{2}\cdot\frac{\sqrt{2}}{2} = \boxed{-\frac{\sqrt{2}}{2}}$$

**Verify.**

**5.** $(\sin x + \cos x)^2 = 1 + \sin 2x$

$$\sin^2 x + 2\sin x\cos x + \cos^2 x$$

$$1 + 2\sin x\cos x$$

$$1 + \sin 2x$$

**6.** $\dfrac{\sec x - 1}{\tan x} = \tan\dfrac{x}{2}$

$$\frac{\frac{1}{\cos x} - \frac{\cos x}{\cos x}}{\frac{\sin x}{\cos x}} \qquad \frac{1-\cos x}{\sin x}$$

$$\frac{\frac{1-\cos x}{\cos x}}{\frac{\sin x}{\cos x}}$$

$$\frac{1-\cos x}{\sin x}$$

**Verify.**

**7.** $\dfrac{2\cos 3x}{\sin 4x - \sin 2x} = \csc x$

$$\frac{2\cos 3x}{2\sin\left(\frac{4x-2x}{2}\right)\cos\left(\frac{4x+2x}{2}\right)}$$

$$\frac{\cancel{2\cos 3x}}{\cancel{2}\sin x\,\cancel{\cos 3x}}$$

$$\frac{1}{\sin x}$$

$$\csc x$$

**Solve. Write <u>all</u> answers in general form.**

**8.** $\sin 2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$

$\cos x\,(2\sin x - 1) = 0$

$\cos x = 0, \quad 2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$$\boxed{x = \frac{\pi}{2} + 2\pi n \quad x = \frac{\pi}{6} + 2\pi n \quad x = \frac{3\pi}{2} + 2\pi n \quad x = \frac{5\pi}{6} + 2\pi n}$$

**9.** $\sin 2x + \sqrt{2}\sin x = 0$

$2\sin x\cos x + \sqrt{2}\sin x = 0$

$\sin x\,(2\cos x + \sqrt{2}) = 0$

$\sin x = 0 \qquad 2\cos x + \sqrt{2} = 0$

$\cos x = -\frac{\sqrt{2}}{2}$

$$\boxed{x = 0 + 2\pi n \quad x = \pi + 2\pi n}$$

$\downarrow$

$$\boxed{x = \pi n}$$

$$\boxed{x = \frac{3\pi}{4} + 2\pi n \quad x = \frac{5\pi}{4} + 2\pi n}$$

**10.** $\sin 4x - \sin 2x = 0$

$2\sin x\cos 3x = 0$

$\sin x = 0 \qquad \cos 3x = 0$

$x = 0 + 2\pi n \qquad 3x = \frac{\pi}{2} + 2\pi n \qquad 3x = \frac{3\pi}{2} + 2\pi n$

$x = \pi + 2\pi n$

$\downarrow$

$$\boxed{x = \pi n} \qquad \boxed{x = \frac{\pi}{6} + \frac{2\pi}{3}n} \qquad \boxed{x = \frac{\pi}{2} + \frac{2\pi}{3}n}$$