Pre-Calculus **HW#3.5** **Name:** Key

The significance of *e* is that many natural events are continuously changing. Whether it is population growth or some sort of decay, nature does not allow events to occur in nice, evenly measured increments. Therefore, exponential growth or decay is often best modeled using the natural number, *e*.

1. Assume that the year 2000 census for a city of 750,000 people reveals an annual increase of 2.3%. Predict the population for the year 2010.

a) Use the formula: $A = P(1+\frac{r}{n})^{nt}$. Assume that n=1.

$750{,}000\left(1+\frac{.023}{1}\right)^{1(10)} = \boxed{941{,}494}$

b) Use the Pert formula

$750{,}000\, e^{(.023 * 10)} = \boxed{943{,}950}$

c) Why are a and b different?

COMPOUNDING ONCE A YEAR VS. COMPOUNDING CONTINUOUSLY

2. The number of people who live on farms in the United States is steadily declining at an estimated rate of about 2% a year (r = -0.02). If there were 5 million people on farms in 2000, use the Pert formula to determine:

a) The number of people on farms in 2010. $5{,}000{,}000\, e^{(-.02)(10)} = \boxed{4{,}093{,}654}$

b) The number of people on farms in 1985.

$5{,}000{,}000\, e^{(-.02)(-15)} = \boxed{6{,}749{,}294}$

c) The number of people on farms in 1800.

$5{,}000{,}000\, e^{(-.02)(-200)} = \boxed{272{,}990{,}750}$

d) Is there anything wrong with the answer in c? Why?

HIGHER THAN THE POPULATION OF THE COUNTRY

MODEL GOES BEYOND ITS USEFULNESS

3. To use the continuous change formula for Carbon-14 dating, the annual rate of decay is –0.000121.

a) Find the percentage of C-14 remaining after 3000 years.

$100\, e^{(-.000121)(3000)} = \boxed{69.6\%}$

b) Find the percentage of C-14 remaining after 40,000 years.

$100\, e^{(-.000121)(40000)} = \boxed{.79\%}$

4. A deep space probe is fitted with a solar panel to provide power for radio communications back to Earth. As the satellite travels farther and farther away from the Sun, eventually the panel will not produce the necessary 5 watts of energy to send signals back. Near the Earth the satellite signal is measured at 25 watts, but it decays by 1% every 20 million miles. Find the signal strength at each planet. $25e^{(-.01)t}$

Planet	**Miles to Earth**	**Signal Strength**
Mars	48.6 million $t = 2.43$	24.4 watts
Jupiter	390.6 million $t = 19.53$	20.6
Saturn	793.7 million $t = 39.685$	16.8
Uranus	1690 million $t = 84.5$	10.7
Neptune	2701 million $t = 135.05$	6.5
Pluto	3573 million $t = 178.65$	4.2

5. Solve the following equations:

a) $6\ln(4x)-1=14$

$6\ln 4x = 15$

$\ln 4x = \frac{15}{6}$

$4x = e^{\frac{15}{6}}$

$x = \frac{e^{\frac{15}{6}}}{4}$

$\boxed{x = 3.05}$

b) $2\log_2 2x = 19$

$\log_2 2x = \frac{19}{2}$

$2x = 2^{\frac{19}{2}}$

$x = \frac{2^{\frac{19}{2}}}{2}$

$\boxed{x = 362.04}$

c) $\log(3x+7)+\log(x-2)=1$

$\log(3x+7)(x-2) = 1$

$3x^2 + x - 14 = 10^1$

$3x^2 + x - 24 = 0$

$(3x-8)(x+3) = 0$

$\boxed{x = \frac{8}{3}}$ or $x \neq -3$

d) $3^{x-2} = 9^{x+3}$

$\log_3 3^{x-2} = \log_3 9^{x+3}$

$x-2 = (x+3)2$

$x-2 = 2x+6$

$\boxed{-8 = x}$

e) $4^{2x+1} = 3^{2x}$

$2x+1 = \log_4 3^{2x}$

$2x+1 = 2x(.792)$

$2x+1 = 1.585x$

$\boxed{x = -2.41}$

f) $3^{3x-2} = 2^x$

$3x-2 = \log_3 2^x$

$3x-2 = x(.631)$

$2.369x = 2$

$\boxed{x = .84}$