

$$9. \frac{x}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2}$$

$$x = A(x-2) + B(x-3)$$

$$x = Ax - 2A + Bx - 3B$$

$$x = (A+B)x + (-2A-3B)$$

$$A+B = 1 \rightarrow A = 3$$

$$-2A-3B = 0 \rightarrow B = -2$$

$$\boxed{\frac{3}{x-3} + \frac{-2}{x-2}}$$

$$12. \frac{5x-1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

$$5x-1 = A(x+1) + B(x-2)$$

$$= Ax + A + Bx - 2B$$

$$= (A+B)x + (A-2B)$$

$$A+B = 5 \rightarrow A = 3$$

$$A-2B = -1 \rightarrow B = 2$$

$$\boxed{\frac{3}{x-2} + \frac{2}{x+1}}$$

$$15. \frac{4}{2x^2-5x-3} = \frac{4}{(2x+1)(x-3)} = \frac{A}{2x+1} + \frac{B}{x-3}$$

$$4 = A(x-3) + B(2x+1)$$

$$= Ax - 3A + 2Bx + B$$

$$= (A+2B)x + (-3A+B)$$

$$A+2B = 0 \rightarrow A = -\frac{B}{2}$$

$$-3A+B = 4 \rightarrow B = \frac{4}{7}$$

$$\boxed{\frac{-8}{7(2x+1)} + \frac{4}{7(x-3)}}$$

$$18. \frac{4x^2-5x-15}{x(x+1)(x-5)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-5}$$

$$4x^2-5x-15 = A(x+1)(x-5) + Bx(x-5) + Cx(x+1)$$

$$= A(x^2-4x-5) + Bx^2-5Bx + Cx^2+Cx$$

$$= Ax^2-4Ax-5A + Bx^2-5Bx + Cx^2+Cx$$

$$= (A+B+C)x^2 + (-4A-5B+C)x + (-5A+C)$$

$$A+B+C = 4$$

$$-4A-5B+C = -5$$

$$-5A+C = -15$$

$$A = 3$$

$$B = -1$$

$$C = 2$$

$$\boxed{\frac{3}{x} + \frac{-1}{x+1} + \frac{2}{x-5}}$$

$$21. \frac{6x-11}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$

$$6x-11 = A(x-1) + B$$

$$= Ax - A + B$$

$$= Ax + (-A+B)$$

$$\boxed{\frac{6}{x-1} + \frac{-5}{(x-1)^2}}$$

$$A = 6$$

$$-A+B = -11 \rightarrow B = -5$$

$$24. \frac{2x^2 + 8x + 3}{(x+1)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

$$A = 2$$

$$2A + B = 8$$

$$A + B + C = 3$$

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$$A = 2$$

$$B = +4$$

$$C = -3$$

$$\begin{aligned} 2x^2 + 8x + 3 &= A(x+1)^2 + B(x+1) + C \\ &= A(x^2 + 2x + 1) + Bx + B + C \\ &= Ax^2 + 2Ax + A + Bx + B + C \\ &= Ax^2 + (2A + B)x + (A + B + C) \end{aligned}$$

$$\boxed{\frac{2}{x+1} + \frac{4}{(x+1)^2} - \frac{3}{(x+1)^3}}$$

$$27. \frac{x^2}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$$

$$x^2 = 2x + 1$$

$$\begin{aligned} x^2 &= A(x-1)(x+1) + B(x+1) + C(x-1)^2 \\ &= Ax^2 - A + Bx + B + Cx^2 - 2Cx + C \\ &= (A+C)x^2 + (B-2C)x + (-A+B+C) \end{aligned}$$

$$A + C = 1$$

$$B - 2C = 0$$

$$-A + B + C = 0$$

$$A = \frac{3}{4}$$

$$B = \frac{1}{2}$$

$$C = \frac{1}{4}$$

$$\boxed{\frac{3}{4(x-1)} + \frac{1}{2(x-1)^2} + \frac{1}{4(x+1)}}$$