

HW # 7.3-3 p. 745 # 30-42 3x

$$30. \frac{5x^2 - 9x + 19}{(x-4)(x^2+5)} = \frac{A}{x-4} + \frac{Bx+C}{x^2+5}$$

$$\begin{aligned} 5x^2 - 9x + 19 &= A(x^2+5) + (Bx+C)(x-4) \\ &= Ax^2 + 5A + Bx^2 - 4Bx + Cx - 4C \\ &= (A+B)x^2 + (-4B+C)x + (5A-4C) \end{aligned}$$

$$\begin{aligned} A+B &= 5 \\ -4B+C &= -9 \\ 5A-4C &= 19 \end{aligned} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 5 \\ 0 & -4 & 1 & -9 \\ 5 & 0 & -4 & 19 \end{bmatrix} \quad A=3, B=2, C=-1$$

$$\boxed{\frac{3}{x-4} + \frac{2x-1}{x^2+5}}$$

$$33. \frac{x+4}{x^2(x^2+4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4}$$

$$\begin{aligned} x+4 &= Ax(x^2+4) + B(x^2+4) + (Cx+D)x^2 \\ &= Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + Dx^2 \\ &= (A+C)x^3 + (B+D)x^2 + 4Ax + 4B \end{aligned}$$

$$\begin{aligned} A+C &= 0 \\ B+D &= 0 \\ 4A &= 1 \\ 4B &= 4 \end{aligned} \rightarrow \begin{aligned} A &= \frac{1}{4} \\ B &= 1 \\ C &= -\frac{1}{4} \\ D &= -1 \end{aligned}$$

$$\boxed{\frac{1}{4x} + \frac{1}{x^2} + \frac{-x-4}{4(x^2+4)}}$$

or

$$\boxed{\frac{1}{4x} + \frac{1}{x^2} + \frac{-\frac{1}{4}x-1}{x^2+4}}$$

$$36. \frac{3x^2 - 2x + 8}{x^3 + 2x^2 + 4x + 8} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$$

$$\frac{x^2(x+2) + 4(x+2)}{(x+2)(x^2+4)}$$

$$\begin{aligned} 3x^2 - 2x + 8 &= A(x^2+4) + (Bx+C)(x+2) \\ &= Ax^2 + 4A + Bx^2 + 2Bx + Cx + 2C \\ &= (A+B)x^2 + (2B+C)x + (4A+2C) \end{aligned}$$

$$\begin{aligned} A+B &= 3 \\ 2B+C &= -2 \\ 4A+2C &= 8 \end{aligned} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 2 & 1 & -2 \\ 4 & 0 & 2 & 8 \end{bmatrix} \quad \begin{aligned} A &= 3 \\ B &= 0 \\ C &= -2 \end{aligned}$$

$$\boxed{\frac{3}{x+2} + \frac{-2}{x^2+4}}$$

$$(39) \frac{x^3 - 4x^2 + 9x - 5}{(x^2 - 2x + 3)^2} = \frac{Ax + B}{x^2 - 2x + 3} + \frac{Cx + D}{(x^2 - 2x + 3)^2}$$

$$\begin{aligned} x^3 - 4x^2 + 9x - 5 &= (Ax + B)(x^2 - 2x + 3) + Cx + D \\ &= Ax^3 - 2Ax^2 + 3Ax + Bx^2 - 2Bx + 3B + Cx + D \\ &= Ax^3 + (-2A + B)x^2 + (3A - 2B + C)x + (3B + D) \end{aligned}$$

$$A = 1$$

$$-2A + B = -4$$

$$3A - 2B + C = 9$$

$$D = -5$$

$$\rightarrow A = 1, B = -2, C = 2, D = -5$$

$$\boxed{\frac{x-2}{x^2-2x+3} + \frac{2x-5}{(x^2-2x+3)^2}}$$

$$(42) \frac{3x-5}{x^3-1} = \frac{3x-5}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$\begin{aligned} 3x-5 &= A(x^2+x+1) + (Bx+C)(x-1) \\ &= Ax^2 + Ax + A + Bx^2 + Cx + Bx - C \\ &= (A+B)x^2 + (A-B+C)x + (A-C) \end{aligned}$$

$$A+B = 0$$

$$A-B+C = 3$$

$$A-C = -5$$

$$A = -\frac{2}{3}$$

$$B = \frac{2}{3}$$

$$C = \frac{13}{3}$$

$$\boxed{\frac{-2}{3(x-1)} + \frac{2x+13}{3(x^2+x+1)}}$$

or

$$\boxed{\frac{-\frac{2}{3}}{3(x-1)} + \frac{\frac{2}{3}x + \frac{13}{3}}{x^2+x+1}}$$