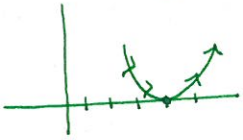
Pre-Calculus **HW#9-R** **Name:** Key

Graph each parametric equation, showing orientation, then eliminate the parameter to show the corresponding rectangular equation.

1. $x = t+4$, $y = t^2$

$x = t+4 \rightarrow t = x-4$

$y = (x-4)^2$

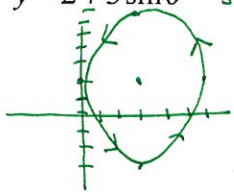


t	0	1
x	4	5
y	0	1

2. $x = 3+3\cos\theta$, $y = 2+5\sin\theta$

$\cos\theta = \frac{x-3}{3}$, $\sin\theta = \frac{y-2}{5}$

$\frac{(x-3)^2}{9} + \frac{(y-2)^2}{25} = 1$



θ	0	$\frac{\pi}{2}$
x	6	3
y	2	7

Write in standard rectangular form and graph. Label the indicated parts.

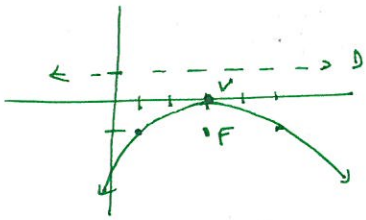
3. $x^2 - 6x + 4y + 9 = 0$ label vertex, focus, directrix

$x^2 - 6x + 9 = -4y - 9 + 9$

$(x-3)^2 = -4y$

$4p = -4$

$p = -1$

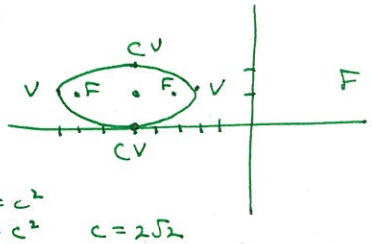


4. $x^2 + 9y^2 + 10x - 18y + 25 = 0$ label vertices, foci

$x^2 + 10x + 25 + 9(y^2 - 2y + 1) = -25 + 25 + 9$

$(x+5)^2 + 9(y-1)^2 = 9$

$\frac{(x+5)^2}{9} + \frac{(y-1)^2}{1} = 1$



$F: (-5 \pm 2\sqrt{2}, 1)$

$9 - 1 = c^2$

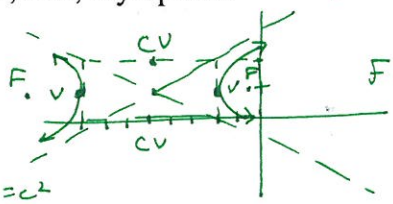
$8 = c^2$ $c = 2\sqrt{2}$

5. $x^2 - 9y^2 + 10x + 18y + 7 = 0$ label vertices, foci, asymptotes

$x^2 + 10x + 25 - 9(y^2 - 2y + 1) = -7 + 25 - 9$

$(x+5)^2 - 9(y-1)^2 = 9$

$\frac{(x+5)^2}{9} - \frac{(y-1)^2}{1} = 1$



$F: (-5 \pm \sqrt{10}, 1)$

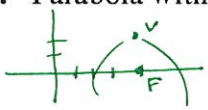
$A: y = 1 \pm \frac{1}{3}(x+5)$

$9 + 1 = c^2$

$\sqrt{10} = c$

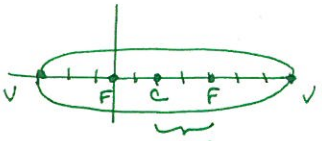
Write each equation in standard rectangular form.

6. Parabola with vertex (4, 2) and focus (4, 0).



$(x-4)^2 = -8(y-2)$

7. Ellipse with vertices (-3, 0) and (7, 0) and foci (0, 0) and (4, 0).



$c = 2$

$a = 5$

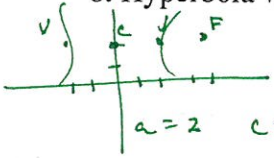
$a^2 - b^2 = c^2$

$25 - b^2 = 4$

$b^2 = 21$

$\frac{(x-2)^2}{25} + \frac{y^2}{21} = 1$

8. Hyperbola with vertices (2, 2) and (-2, 2) and foci (4, 2) and (-4, 2).



$a^2+b^2=c^2$

$4+b^2=16$

$b^2=12$

$$\frac{x^2}{4}-\frac{(y-2)^2}{12}=1$$

Classify and find the angle of rotation.

9. $2x^2+4xy+5y^2+3x-4y-20=0$ $B^2-4AC = 4^2-4(2)(5) = 16-40$ NEGATIVE

$\tan 2\theta = \frac{4}{2-5} = \frac{4}{-3}$ $2\theta = \tan^{-1}(-\frac{4}{3})$ $\theta = 63.4^\circ$ ELLIPSE

10. $x^2-6xy-5y^2+4x-22=0$

$B^2-4AC = 36-4(1)(-5)$ POSITIVE HYPERBOLA $\tan 2\theta = \frac{-6}{1+5} = \frac{-6}{6}$ $2\theta = -\frac{\pi}{4}+\pi = \frac{3\pi}{4}$ $\theta = \frac{3\pi}{8}$ or 67.5°

Classify each of the following as: ellipse, parabola, or hyperbola.

11. $r=\dfrac{4}{6-6\cos\theta}$ $\frac{\frac{4}{6}}{1-\frac{6}{6}\cos\theta}$ PARABOLA

12. $r=\dfrac{2}{3+5\sin\theta}$ $\frac{\frac{2}{3}}{1+\frac{5}{3}\sin\theta}$ HYPERBOLA

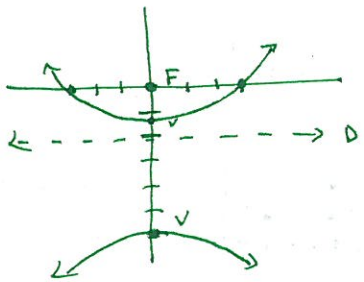
13. $r=\dfrac{5}{3-\sin\theta}$ $\frac{\frac{5}{3}}{1-\frac{1}{3}\sin\theta}$ ELLIPSE

14. $r=\dfrac{6}{12+3\cos\theta}$ $\frac{\frac{6}{12}}{1+\frac{3}{12}\cos\theta}$ ELLIPSE

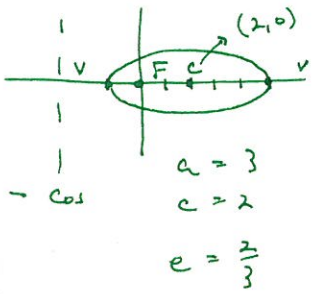
15. Graph. Include the focus and directrix on the graph.

$r=\dfrac{6}{2-3\sin\theta} = \dfrac{3}{1-\frac{3}{2}\sin\theta}$ $e=\frac{3}{2}$ → HYPERBOLA $ep=3$ $p=2$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r	3	-6	3	$\frac{6}{5}$



16. Find a polar equation of an ellipse with the focus at the pole and vertices at (5,0) and $(1,\pi)$



$r=\dfrac{\frac{2}{3}p}{1-\frac{2}{3}\cos\theta}$

$r=\dfrac{2p}{3-2\cos\theta}$

$1=\dfrac{2p}{3-2\cos\pi}$

$5=2p$

$p=\frac{5}{2}$

$$r=\frac{5}{3-2\cos\theta}$$