Key

**2nd Semester Review- Pre-Calculus** **Chapter 9**

**1.** Identify the focus, vertex and directrix of the parabola $x^2-6x-4y+1=0$.

$x^2-6x+9 = 4y-1+9$

$(x-3)^2 = 4(y+2)$

V: (3, -2)
F: (3, -1)
D: $y=-3$

**2.** Identify the center, foci, vertices and eccentricity of the ellipse
$x^2+4y^2-2x+32y+61=0$

$x^2-2x+1+4(y^2+8y+16) = -61+1+64$

$(x-1)^2+4(y+4)^2 = 4$

$\frac{(x-1)^2}{4}+\frac{(y+4)^2}{1}=1$

C: (1, -4)
V: (3, -4) (-1, -4)
F: $(1\pm\sqrt{3}, -4)$
$e=\frac{\sqrt{3}}{2}$

**3.** Find the center, vertices, and foci of the hyperbola $16y^2-x^2-6x-128y+231=0$.

$16(y^2-8x+16)-(x^2+6x+9) = -231+256-9$

$16(y-4)^2-(x+3)^2=16$

$\frac{(y-4)^2}{1}-\frac{(x+3)^2}{16}=1$

C: (-3, 4)
V: (-3, 5) (-3, 3)
F: $(-3, 4\pm\sqrt{17})$

**4.** Classify each conic:

a) $6x^2-2xy+y^2=0$

$4-4\cdot6\cdot1$ = negative

ELLIPSE

b) $x^2+4xy+4y^2-x-y+17=0$

$16-4\cdot1\cdot4=0$

PARABOLA

**5.** Determine the type of conic and the angle of rotation for $5x^2+12xy+5y^2-10=0$.

$144-4\cdot5\cdot5$ = positive

HYPERBOLA

$\tan 2\theta = \frac{12}{5-5}$ = undefined

$2\theta = 90°$

$\theta = 45°$ or $\frac{\pi}{4}$

**6.** Convert to rectangular coordinates.

$(\sqrt{2}, \frac{3\pi}{4})$ $x=r\cos\theta$ $y=r\sin\theta$

(-1, 1)

**7.** Convert to polar coordinates.

$(\sqrt{3}, -1)$ $r=\sqrt{3+1}=2$ $\tan\theta=\frac{-1}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$ Q IV

$(2, \frac{11\pi}{6})$

**8.** Convert to polar form: $4x-3y=12$.

$4r\cos\theta-3r\sin\theta=12$

$r(4\cos\theta-3\sin\theta)=12$

$r=\frac{12}{4\cos\theta-3\sin\theta}$

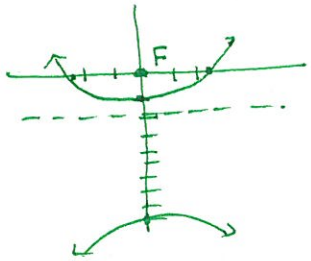
**9.** Convert to rectangular form: $r=5\cos\theta$. $r^2=5\cos\theta\cdot r$ $\rightarrow$ $x^2+y^2=5x$

**10.** Give the eccentricity and name the type of conic: $r=\frac{3}{6-6\cos\theta}$.

$e=1$ PARABOLA

$r=\frac{\frac{1}{2}}{1-1\cos\theta}$

**11.** Sketch the graph of this polar conic: $r = \dfrac{9}{4-5\sin\theta} = \dfrac{\frac{9}{4}}{1-\frac{5}{4}\sin\theta}$ Hyperbola

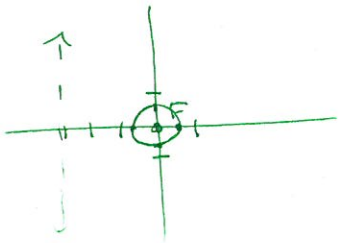



$ep = \frac{9}{4}$

$p = \frac{9}{5}$

| $\theta$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ |
|---|---|---|---|---|
| $r$ | $\frac{9}{4}$ | $-9$ | $\frac{9}{4}$ | $1$ |

**12.** Sketch the graph of the polar conic: $r = \dfrac{3}{6-\cos\theta} = \dfrac{\frac{1}{2}}{1-\frac{1}{6}\cos\theta}$ Ellipse

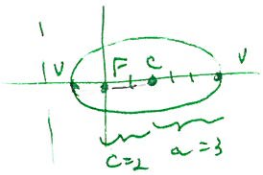



$ep = \frac{1}{2}$

$p = 3$

| $\theta$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ |
|---|---|---|---|---|
| $r$ | $\frac{3}{5}$ | $\frac{1}{2}$ | $\frac{3}{7}$ | $\frac{1}{2}$ |

**13.** Find a polar equation of the ellipse with its vertices at $(5,0),(1,\pi)$ and focus at the pole.




$c = 2 \quad a = 3$

$e = \frac{2}{3}$

$r = \dfrac{\frac{2}{3}p}{1-\frac{2}{3}\cos\theta}$

$r = \dfrac{2p}{3-2\cos\theta}$

$1 = \dfrac{2p}{3-2\cos\pi}$

$p = \frac{5}{2}$

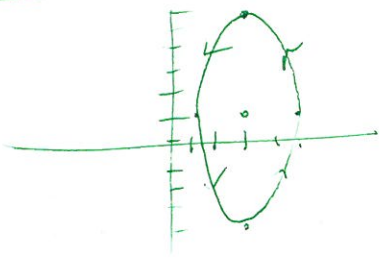
$$r = \frac{5}{3-2\cos\theta}$$

**Sketch the curve represented by the parametric equations including orientation, then eliminate the parameter and write the corresponding rectangular equation.**

**14.** $x = 3-2\sin\theta$ $\quad \sin\theta = \frac{x-3}{-2}$

$y = 1+5\cos\theta$ $\quad \cos\theta = \frac{y-1}{5}$

$$\frac{(x-3)^2}{4} + \frac{(y-1)^2}{25} = 1$$



**15.** $x = 4+t$ $\quad t = x-4$

$y = \frac{1}{2}t$ $\quad y = \frac{1}{2}(x-4)$

$$y = \frac{1}{2}x - 2$$