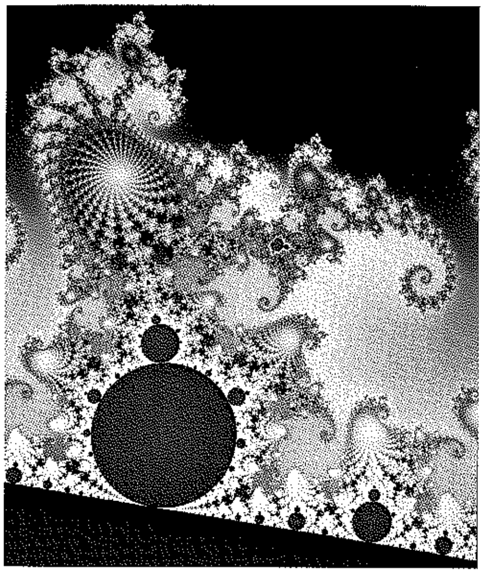
BLITZER BONUS

The Patterns of Chaos



A magnification of the Mandelbrot set. R. F. Voss, "29-Fold M-set Seahorse" computer-generated image.

"The single most compelling reason to explore the world of mathematics is that it is beautiful, and pondering its intriguing ideas is great fun. I'm constantly perplexed by how many people do not believe this, yet over 50,000 professional mathematicians in America practice their trade with enthusiasm and fervor."

Calvin Clawson, *Mathematical Mysteries*, Plenum Press, 1996

One of the new frontiers of mathematics suggests that there is an underlying order in things that appear to be random, such as the hiss and crackle of background noises as you tune a radio. Irregularities in the heartbeat, some of them severe enough to cause a heart attack, or irregularities in our sleeping patterns, such as insomnia, are examples of chaotic behavior. Chaos in the mathematical sense does not mean a complete lack of form or arrangement. In mathematics, chaos is used to describe something that appears to be random but is not actually random. The patterns of chaos appear in images like the one shown on the left, called the Mandelbrot set. Magnified portions of this image yield repetitions of the original structure, as well as new and unexpected patterns. The Mandelbrot set transforms the hidden structure of chaotic events into a source of wonder and inspiration.

EXERCISE SET 1.1

STUDY TIP

Researchers say the mind can be strengthened, just like your muscles, with regular training and rigorous practice. Think of the book's exercise sets as brain calisthenics. If you're feeling a bit sluggish before any of your mental workouts, try this warmup:

> In the list below, say the color the word is printed in, not the word itself. Once you can do this in 15 seconds without an error, the warmup is over and it's time to move on to the assigned exercises.
> Blue Yellow Red Green Yellow Green Blue Red Yellow Red

• Practice Exercises

In Exercises 1–30, identify a pattern in each list of numbers. Then use this pattern to find the next number. (More than one pattern might exist, so it is possible that there is more than one correct answer.)

1. 8, 12, 16, 20, 24, ____

2. 19, 24, 29, 34, 39, ____

3. 37, 32, 27, 22, 17, ____

4. 33, 29, 25, 21, 17, ____

5. 3, 9, 27, 81, 243, ____

6. 2, 8, 32, 128, 512, ____

7. 1, 2, 4, 8, 16, ____

8. 1, 5, 25, 125, ____

9. 1, 4, 1, 8, 1, 16, 1, ____

10. 1, 4, 1, 7, 1, 10, 1, ____

11. 4, 2, 0, −2, −4, ____

12. 6, 3, 0, −3, −6, ____

13. $\frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \frac{1}{14}, \frac{1}{18}$, ____

14. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, ____

15. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$, ____

16. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$, ____

17. 3, 7, 12, 18, 25, 33, ____

18. 2, 5, 9, 14, 20, 27, ____

19. 3, 6, 11, 18, 27, 38, ____

20. 2, 5, 10, 17, 26, 37, ____

21. 3, 7, 10, 17, 27, 44, ____

22. 2, 5, 7, 12, 19, 31, ____

23. 2, 7, 12, 5, 10, 15, 8, 13, ____

24. 3, 9, 15, 5, 11, 17, 7, 13, ____

25. 3, 6, 5, 10, 9, 18, 17, 34, ____

26. 2, 6, 5, 15, 14, 42, 41, 123, ____

27. 64, −16, 4, −1, ____

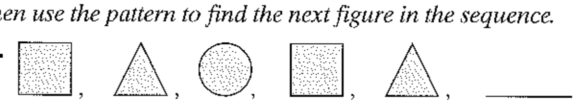
28. 125, −25, 5, −1, ____

29. $(6, 2), (0, -4), \left(7\frac{1}{2}, 3\frac{1}{2}\right), (2, -2), (3,$ ____$)$

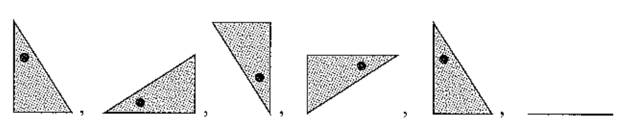
30. $\left(\frac{2}{3}, \frac{4}{9}\right), \left(\frac{1}{5}, \frac{1}{25}\right), (7, 49), \left(-\frac{5}{6}, \frac{25}{36}\right), \left(-\frac{4}{7},\right.$ ____$\left.\right)$

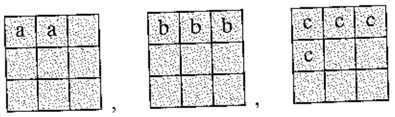
In Exercises 31–34, identify a pattern in each sequence of figures. Then use the pattern to find the next figure in the sequence.

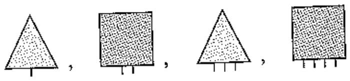
31.



32.



33.  , ______

34.  , ______

Exercises 35–38 describe procedures that are to be applied to numbers. In each exercise,

a. *Repeat the procedure for four numbers of your choice. Write a conjecture that relates the result of the process to the original number selected.*

b. *Represent the original number by the variable n and use deductive reasoning to prove the conjecture in part (a).*

35. Select a number. Multiply the number by 4. Add 8 to the product. Divide this sum by 2. Subtract 4 from the quotient.

36. Select a number. Multiply the number by 3. Add 6 to the product. Divide this sum by 3. Subtract the original selected number from the quotient.

37. Select a number. Add 5. Double the result. Subtract 4. Divide by 2. Subtract the original selected number.

38. Select a number. Add 3. Double the result. Add 4. Divide by 2. Subtract the original selected number.

In Exercises 39–44, use inductive reasoning to predict the next line in each sequence of computations. Then use a calculator or perform the arithmetic by hand to determine whether your conjecture is correct.

39.

$$1 + 2 = \frac{2 \times 3}{2}$$
$$1 + 2 + 3 = \frac{3 \times 4}{2}$$
$$1 + 2 + 3 + 4 = \frac{4 \times 5}{2}$$
$$1 + 2 + 3 + 4 + 5 = \frac{5 \times 6}{2}$$

40.

$$3 + 6 = \frac{6 \times 3}{2}$$
$$3 + 6 + 9 = \frac{9 \times 4}{2}$$
$$3 + 6 + 9 + 12 = \frac{12 \times 5}{2}$$
$$3 + 6 + 9 + 12 + 15 = \frac{15 \times 6}{2}$$

41.

$$1 + 3 = 2 \times 2$$
$$1 + 3 + 5 = 3 \times 3$$
$$1 + 3 + 5 + 7 = 4 \times 4$$
$$1 + 3 + 5 + 7 + 9 = 5 \times 5$$

42.

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} = \frac{2}{3}$$
$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} = \frac{3}{4}$$
$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} = \frac{4}{5}$$

43.

$$9 \times 9 + 7 = 88$$
$$98 \times 9 + 6 = 888$$
$$987 \times 9 + 5 = 8888$$
$$9876 \times 9 + 4 = 88{,}888$$

44.

$$1 \times 9 - 1 = 8$$
$$21 \times 9 - 1 = 188$$
$$321 \times 9 - 1 = 2888$$
$$4321 \times 9 - 1 = 38{,}888$$

• Practice Plus

In Exercises 45–46, use inductive reasoning to predict the next line in each sequence of computations. Then use a calculator or perform the arithmetic by hand to determine whether your conjecture is correct.

45.

$$33 \times 3367 = 111{,}111$$
$$66 \times 3367 = 222{,}222$$
$$99 \times 3367 = 333{,}333$$
$$132 \times 3367 = 444{,}444$$

46.

$$1 \times 8 + 1 = 9$$
$$12 \times 8 + 2 = 98$$
$$123 \times 8 + 3 = 987$$
$$1234 \times 8 + 4 = 9876$$
$$12{,}345 \times 8 + 5 = 98{,}765$$

47. Study the pattern in these examples:

$$a^2 \# a^4 = a^{10} \quad a^3 \# a^2 = a^7 \quad a^5 \# a^3 = a^{11}.$$

Select the equation that describes the pattern.

a. $a^x \# a^y = a^{2x+y}$

b. $a^x \# a^y = a^{x+2y}$

c. $a^x \# a^y = a^{x+y+4}$

d. $a^x \# a^y = a^{xy+2}$

48. Study the pattern in these examples:

$$a^5 * a^3 * a^2 = a^5 \quad a^3 * a^7 * a^2 = a^6 \quad a^2 * a^4 * a^8 = a^7.$$

Select the equation that describes the pattern.

a. $a^x * a^y * a^z = a^{x+y+z}$

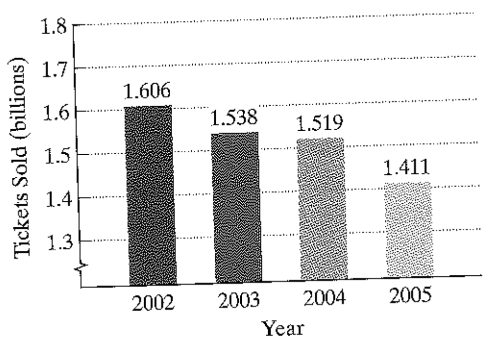
b. $a^x * a^y * a^z = a^{\frac{xyz}{2}}$

c. $a^x * a^y * a^z = a^{\frac{x+y+z}{2}}$

d. $a^x * a^y * a^z = a^{\frac{xy}{2}+z}$

49. The trend in the data displayed by the graph leaves little for Hollywood executives (or even King Kong) to beat their chests about. Study the pattern, or trend, shown by the data. Then select the expression that *best* describes the number of movie tickets sold, in billions, n years after 2002.

Theater Attendance: Number of Movie Tickets Sold in the United States



Source: Entertainment Weekly

a. $1.6 + 0.06n$

b. $1.6 + 1.06n$

c. $1.6 - 0.06n$

d. $1.6 - 1.06n$

58. Describe what is meant by deductive reasoning. Give an example.

59. Give an example of a decision that you made recently in which the method of reasoning you used to reach the decision was induction. Describe your reasoning process.

60. The April 26, 2004 issue of *Time* magazine is a special issue on the lives and ideas of the world's most influential people. Consult this issue of *Time* and select one of the people who used inductive or deductive reasoning in his or her work.

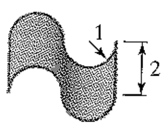
a. Explain how the person you selected used one of these types of reasoning.

b. Explain why you selected this particular person.

• Critical Thinking Exercises

61. If $(6-2)^2 = 36 - 24 + 4$ and $(8-5)^2 = 64 - 80 + 25$, use inductive reasoning to write a compatible expression for $(11-7)^2$.

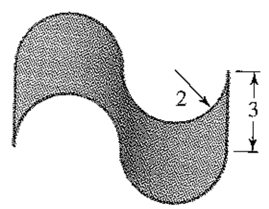
62. Study the first three figures. Then use inductive reasoning to determine the area of the same type of figure with a radius of 9 and a height of 10.



Radius of each semicircle = 1

Height = 2

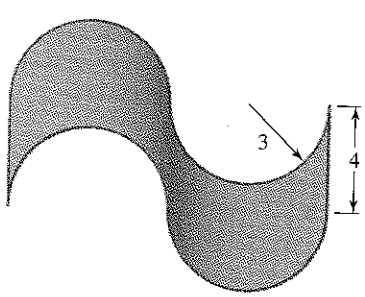
Area = 8 square units



Radius of each semicircle = 2

Height = 3

Area = 24 square units



Radius of each semicircle = 3

Height = 4

Area = 48 square units

63. Write a list of numbers that has two patterns so that the next number in the list can be 15 or 20.

64. a. Repeat the following procedure with at least five people. Write a conjecture that relates the result of the procedure to each person's birthday.

Take the number of the month of your birthday (January = 1, February = 2, ..., December = 12), multiply by 5, add 6, multiply this sum by 4, add 9, multiply this new sum by 5, and add the number of the day on which you were born. Finally, subtract 165.

b. Let M represent the month number and let D represent the day number of any person's birthday. Use deductive reasoning to prove your conjecture in part (a).

• Technology Exercises

65. a. Use a calculator to find 6×6, 66×66, 666×666, and 6666×6666.

b. Describe a pattern in the numbers being multiplied and the resulting products.

c. Use the pattern to write the next two multiplications and their products. Then use your calculator to verify these results.

d. Is this process an example of inductive or deductive reasoning? Explain your answer.

66. a. Use a calculator to find 3367×3, 3367×6, 3367×9, and 3367×12.

b. Describe a pattern in the numbers being multiplied and the resulting products.

c. Use the pattern to write the next two multiplications and their products. Then use your calculator to verify these results.

d. Is this process an example of inductive or deductive reasoning? Explain your answer.

• Group Exercise

67. Stereotyping refers to classifying people, places, or things according to common traits. Prejudices and stereotypes can function as assumptions in our thinking, appearing in inductive and deductive reasoning. For example, it is not difficult to find inductive reasoning that results in generalizations such as these, as well as deductive reasoning in which these stereotypes serve as assumptions:

School has nothing to do with life.

Intellectuals are nerds.

People on welfare are lazy.

Each group member should find one example of inductive reasoning and one example of deductive reasoning in which stereotyping occurs. Upon returning to the group, present each example and then describe how the stereotyping results in faulty conjectures or prejudging situations and people.