

**SOLUTION** Let  $A$  = the set of students willing to donate blood and  $B$  = the set of students willing to serve breakfast. We are interested in how many students were willing to donate blood or serve breakfast. Thus, we need to determine  $n(A \cup B)$ .

number of blood donors or breakfast servers      number of blood donors      number of breakfast servers      number of blood donors and breakfast servers

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 490 + 340 - 120 \\ &= 830 - 120 \\ &= 710 \end{aligned}$$

We see that 710 students were willing to donate blood or serve a free breakfast.  $\square$



The admissions department at a college looked at the registration of 500 of its students and found the following results: 244 students were registered in a mathematics class, 230 students were registered in an English class, and 89 students were registered in a math class and an English class. How many students were registered in a math class or an English class?

## EXERCISE SET 2.3

### • Practice Exercises

In Exercises 1–4, describe a universal set  $U$  that includes all elements in the given sets. Answers may vary.

- $A = \{\text{Bach, Mozart, Beethoven}\}$   
 $B = \{\text{Brahms, Schubert}\}$
- $A = \{\text{William Shakespeare, Charles Dickens}\}$   
 $B = \{\text{Mark Twain, Robert Louis Stevenson}\}$
- $A = \{\text{Pepsi, Sprite}\}$   
 $B = \{\text{Coca Cola, Seven-Up}\}$
- $A = \{\text{Acura RSX, Toyota Camry, Mitsubishi Lancer}\}$   
 $B = \{\text{Dodge Ram, Chevrolet Impala}\}$

In Exercises 5–8, let  $U = \{a, b, c, d, e, f, g\}$ ,  $A = \{a, b, f, g\}$ ,  $B = \{c, d, e\}$ ,  $C = \{a, g\}$ , and  $D = \{a, b, c, d, e, f\}$ . Use the roster method to write each of the following sets.

- $A'$
- $B'$
- $C'$
- $D'$

In Exercises 9–12, let  $U = \{1, 2, 3, 4, \dots, 20\}$ ,  $A = \{1, 2, 3, 4, 5\}$ ,  $B = \{6, 7, 8, 9\}$ ,  $C = \{1, 3, 5, 7, \dots, 19\}$ , and  $D = \{2, 4, 6, 8, \dots, 20\}$ . Use the roster method to write each of the following sets.

- $A'$
- $B'$
- $C'$
- $D'$

In Exercises 13–16, let  $U = \{1, 2, 3, 4, \dots\}$ ,  $A = \{1, 2, 3, 4, \dots, 20\}$ ,  $B = \{1, 2, 3, 4, \dots, 50\}$ ,  $C = \{2, 4, 6, 8, \dots\}$ , and  $D = \{1, 3, 5, 7, \dots\}$ . Use the roster method to write each of the following sets.

- $A'$
- $B'$
- $C'$
- $D'$

In Exercises 17–40, let

$$\begin{aligned} U &= \{1, 2, 3, 4, 5, 6, 7\} \\ A &= \{1, 3, 5, 7\} \\ B &= \{1, 2, 3\} \\ C &= \{2, 3, 4, 5, 6\} \end{aligned}$$

Find each of the following sets.

- |                        |                        |                        |                        |
|------------------------|------------------------|------------------------|------------------------|
| 17. $A \cap B$         | 18. $B \cap C$         | 19. $A \cup B$         | 20. $B \cup C$         |
| 21. $A'$               | 22. $B'$               | 23. $A' \cap B'$       | 24. $B' \cap C'$       |
| 25. $A \cup C'$        | 26. $B \cup C'$        | 27. $(A \cap C)'$      | 28. $(A \cap B)'$      |
| 29. $A' \cup C'$       | 30. $A' \cup B'$       | 31. $(A \cup B)'$      | 32. $(A \cup C)'$      |
| 33. $A \cup \emptyset$ | 34. $C \cup \emptyset$ | 35. $A \cap \emptyset$ | 36. $C \cap \emptyset$ |
| 37. $A \cup U$         | 38. $B \cup U$         | 39. $A \cap U$         | 40. $B \cap U$         |

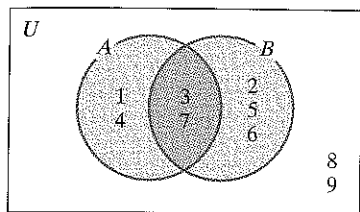
In Exercises 41–66, let

$$\begin{aligned} U &= \{a, b, c, d, e, f, g, h\} \\ A &= \{a, g, h\} \\ B &= \{b, g, h\} \\ C &= \{b, c, d, e, f\} \end{aligned}$$

Find each of the following sets.

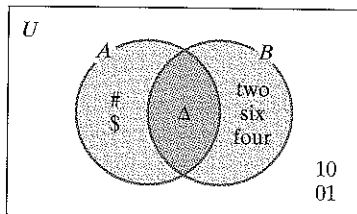
- |                          |                          |                        |                        |
|--------------------------|--------------------------|------------------------|------------------------|
| 41. $A \cap B$           | 42. $B \cap C$           | 43. $A \cup B$         | 44. $B \cup C$         |
| 45. $A'$                 | 46. $B'$                 | 47. $A' \cap B'$       | 48. $B' \cap C'$       |
| 49. $A \cup C'$          | 50. $B \cup C'$          | 51. $(A \cap C)'$      | 52. $(A \cap B)'$      |
| 53. $A' \cup C'$         | 54. $A' \cup B'$         | 55. $(A \cup B)'$      | 56. $(A \cup C)'$      |
| 57. $A \cup \emptyset$   | 58. $C \cup \emptyset$   | 59. $A \cap \emptyset$ | 60. $C \cap \emptyset$ |
| 61. $A \cup U$           | 62. $B \cup U$           | 63. $A \cap U$         | 64. $B \cap U$         |
| 65. $(A \cap B) \cup B'$ | 66. $(A \cup B) \cap B'$ |                        |                        |

In Exercises 67–78, use the Venn diagram to represent each set in roster form.



67.  $A$     68.  $B$     69.  $U$     70.  $A \cup B$   
 71.  $A \cap B$     72.  $A'$     73.  $B'$     74.  $(A \cap B)'$   
 75.  $(A \cup B)'$     76.  $A' \cap B$     77.  $A \cap B'$     78.  $A \cup B'$

In Exercises 79–92, use the Venn diagram to determine each set or cardinality.



79.  $B$     80.  $A$     81.  $A \cup B$   
 82.  $A \cap B$     83.  $n(A \cup B)$     84.  $n(A \cap B)$   
 85.  $n(A')$     86.  $n(B')$     87.  $(A \cap B)'$   
 88.  $(A \cup B)'$     89.  $A' \cap B$     90.  $A \cap B'$   
 91.  $n(U) - n(B)$     92.  $n(U) - n(A)$

Use the formula for the cardinal number of the union of two sets to solve Exercise 93–96.

93. Set  $A$  contains 17 elements, set  $B$  contains 20 elements, and 6 elements are common to sets  $A$  and  $B$ . How many elements are in  $A \cup B$ ?  
 94. Set  $A$  contains 30 elements, set  $B$  contains 18 elements, and 5 elements are common to sets  $A$  and  $B$ . How many elements are in  $A \cup B$ ?  
 95. Set  $A$  contains 8 letters and 9 numbers. Set  $B$  contains 7 letters and 10 numbers. Four letters and 3 numbers are common to both sets  $A$  and  $B$ . Find the number of elements in set  $A$  or set  $B$ .  
 96. Set  $A$  contains 12 numbers and 18 letters. Set  $B$  contains 14 numbers and 10 letters. One number and 6 letters are common to both sets  $A$  and  $B$ . Find the number of elements in set  $A$  or set  $B$ .

**• Practice Plus**

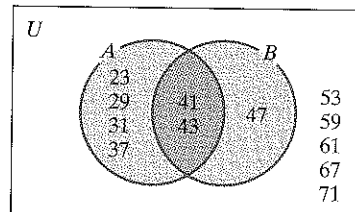
In Exercises 97–104, let

- $U = \{x | x \in \mathbb{N} \text{ and } x < 9\}$   
 $A = \{x | x \text{ is an odd natural number and } x < 9\}$   
 $B = \{x | x \text{ is an even natural number and } x < 9\}$   
 $C = \{x | x \in \mathbb{N} \text{ and } 1 < x < 6\}$

Find each of the following sets.

97.  $A \cup B$     98.  $B \cup C$     99.  $A \cap U$     100.  $A \cup U$   
 101.  $A \cap C'$     102.  $A \cap B'$     103.  $(B \cap C)'$     104.  $(A \cap C)'$

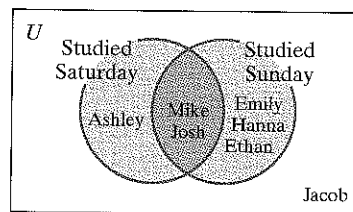
In Exercises 105–108, use the Venn diagram to determine each set or cardinality.



105.  $A \cup (A \cup B)'$   
 106.  $(A' \cap B) \cup (A \cap B)$   
 107.  $n(U)[n(A \cup B) - n(A \cap B)]$   
 108.  $n(A \cap B)[n(A \cup B) - n(A')]$

**• Application Exercises**

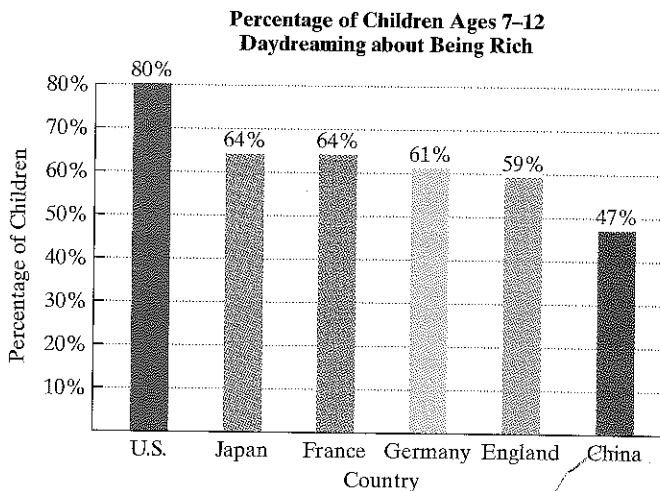
A math tutor working with a small group of students asked each student when he or she had studied for class the previous weekend. Their responses are shown in the Venn diagram.



In Exercises 109–116, use the Venn diagram to list the elements of each set in roster form.

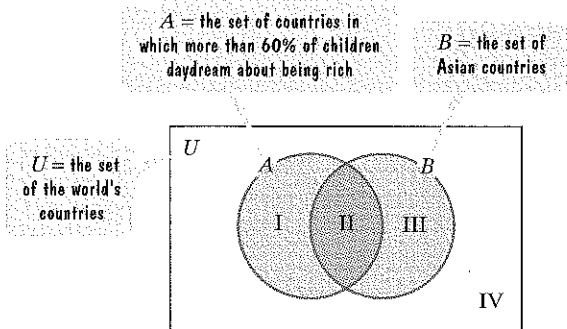
109. The set of students who studied Saturday  
 110. The set of students who studied Sunday  
 111. The set of students who studied Saturday or Sunday  
 112. The set of students who studied Saturday and Sunday  
 113. The set of students who studied Saturday and not Sunday  
 114. The set of students who studied Sunday and not Saturday  
 115. The set of students who studied neither Saturday nor Sunday  
 116. The set of students surveyed by the math tutor

The bar graph shows the percentage of children in the world's leading industrial countries who daydream about being rich.



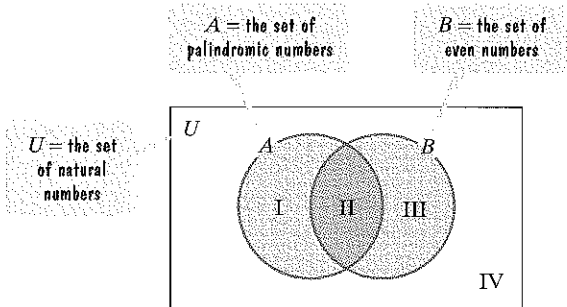
Source: Roper Starch Worldwide for A.B.C. Research

In Exercises 117–122, use the information in the graph on the previous page to place the indicated country in the correct region of the following Venn diagram.



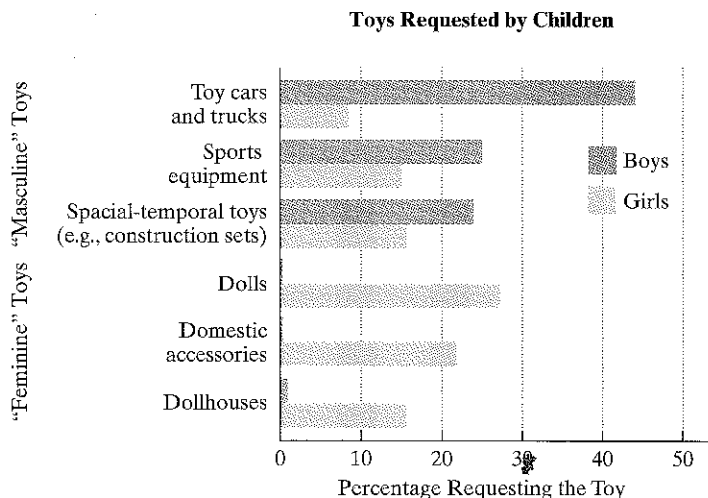
117. U.S.    118. Japan    119. France    120. Germany  
 121. England    122. China

A **palindromic number** is a natural number whose value does not change if its digits are reversed. Examples of palindromic numbers are 11, 454, and 261,162. In Exercises 123–132, use this definition to place the indicated natural number in the correct region of the following Venn diagram.



123. 11    124. 22    125. 15    126. 17  
 127. 454    128. 101    129. 9558    130. 9778  
 131. 9559    132. 9779

As a result of cultural expectations about what is appropriate behavior for each gender, boys and girls differ substantially in their toy preferences. The graph shows the percentage of boys and girls asking for various types of toys in letters to Santa Claus. Use the information in the graph to write each set in Exercises 133–138 in roster form or express the set as  $\emptyset$ .



Source: Richard, J.G., & Simpson, C.H. (1982). Children, gender and social structure: An analysis of the contents of letters to Santa Claus. *Child Development*, 53, 429–436.

133.  $\{x | x \text{ is a toy requested by more than 10\% of the boys}\} \cap \{x | x \text{ is a toy requested by less than 20\% of the girls}\}$
134.  $\{x | x \text{ is a toy requested by fewer than 5\% of the boys}\} \cap \{x | x \text{ is a toy requested by fewer than 20\% of the girls}\}$
135.  $\{x | x \text{ is a toy requested by more than 10\% of the boys}\} \cup \{x | x \text{ is a toy requested by less than 20\% of the girls}\}$
136.  $\{x | x \text{ is a toy requested by fewer than 5\% of the boys}\} \cup \{x | x \text{ is a toy requested by fewer than 20\% of the girls}\}$
137. The set of toys requested by more than 40% of the boys and more than 10% of the girls
138. The set of toys requested by more than 40% of the boys or more than 10% of the girls
139. A winter resort took a poll of its 350 visitors to see which winter activities people enjoyed. The results were as follows: 178 people liked to ski, 154 people liked to snowboard, and 49 people liked to ski and snowboard. How many people in the poll liked to ski or snowboard?
140. A pet store surveyed 200 pet owners and obtained the following results: 96 people owned cats, 97 people owned dogs, and 29 people owned cats and dogs. How many people in the survey owned cats or dogs?

• **Writing in Mathematics**

141. Describe what is meant by a universal set. Provide an example.
142. What is a Venn diagram and how is it used?
143. Describe the Venn diagram for two disjoint sets. How does this diagram illustrate that the sets have no common elements?
144. Describe the Venn diagram for proper subsets. How does this diagram illustrate that the elements of one set are also in the second set?
145. Describe the Venn diagram for two equal sets. How does this diagram illustrate that the sets are equal?
146. Describe the Venn diagram for two sets with common elements. How does the diagram illustrate this relationship?
147. Describe what is meant by the complement of a set.
148. Is it possible to find a set's complement if a universal set is not given? Explain your answer.
149. Describe what is meant by the intersection of sets. Give an example.
150. Describe what is meant by the union of sets. Give an example.
151. Describe how to find the cardinal number of the union of two finite sets.

• **Critical Thinking Exercises**

152. Which one of the following is true?
- $n(A \cup B) = n(A) + n(B)$
  - $A \cap A' = \emptyset$
  - $(A \cup B) \subseteq A$
  - If  $A \subseteq B$ , then  $A \cap B = B$ .