

## BLITZER BONUS

## Blood Types and Venn Diagrams

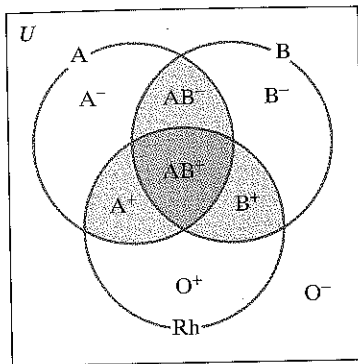


FIGURE 2.23 Human blood types

In the early 1900s, the Austrian immunologist Karl Landsteiner discovered that all blood is not the same. Blood serum drawn from one person often clumped when mixed with the blood cells of another. The clumping was caused by different antigens, proteins, and carbohydrates that trigger antibodies and fight infection. Landsteiner classified blood types based on the presence or absence of the antigens A, B, and Rh in red blood cells. The Venn diagram in Figure 2.23 contains eight regions representing the eight common blood groups.

In the Venn diagram, blood with the Rh antigen is labeled positive and blood lacking the Rh antigen is labeled negative. The region where the three circles intersect represents type  $AB^+$ , indicating that a person with this blood type has the antigens A, B, and Rh. Observe that type O blood (both positive and negative) lacks A and B antigens. Type  $O^-$  lacks all three antigens, A, B, and Rh.

In blood transfusions, the recipient must have all or more of the antigens present in the donor's blood. This discovery rescued surgery patients from random, often lethal, transfusions. This knowledge made the massive blood drives during World War I possible. Eventually, it made the modern blood bank possible as well.

## EXERCISE SET 2.4

## • Practice Exercises

In Exercises 1–12, let

$$U = \{1, 2, 3, 4, 5, 6, 7\}$$

$$A = \{1, 3, 5, 7\}$$

$$B = \{1, 2, 3\}$$

$$C = \{2, 3, 4, 5, 6\}.$$

Find each of the following sets.

- |                                    |                                    |
|------------------------------------|------------------------------------|
| 1. $A \cup (B \cap C)$             | 2. $A \cap (B \cup C)$             |
| 3. $(A \cup B) \cap (A \cup C)$    | 4. $(A \cap B) \cup (A \cap C)$    |
| 5. $A' \cap (B \cup C')$           | 6. $C' \cap (A \cup B')$           |
| 7. $(A' \cap B) \cup (A' \cap C')$ | 8. $(C' \cap A) \cup (C' \cap B')$ |
| 9. $(A \cup B \cup C)'$            | 10. $(A \cap B \cap C)'$           |
| 11. $(A \cup B)' \cap C$           | 12. $(B \cup C)' \cap A$           |

In Exercises 13–24, let

$$U = \{a, b, c, d, e, f, g, h\}$$

$$A = \{a, g, h\}$$

$$B = \{b, g, h\}$$

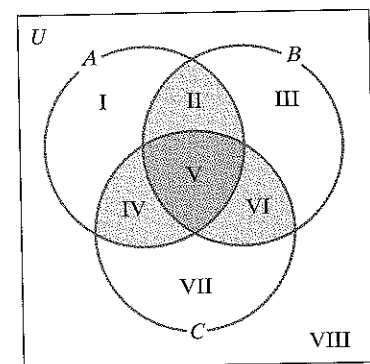
$$C = \{b, c, d, e, f\}.$$

Find each of the following sets.

- |                                  |                                  |
|----------------------------------|----------------------------------|
| 13. $A \cup (B \cap C)$          | 14. $A \cap (B \cup C)$          |
| 15. $(A \cup B) \cap (A \cup C)$ | 16. $(A \cap B) \cup (A \cap C)$ |
| 17. $A' \cap (B \cup C')$        | 18. $C' \cap (A \cup B')$        |

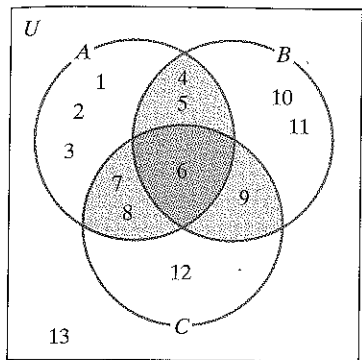
- |                                     |                                     |
|-------------------------------------|-------------------------------------|
| 19. $(A' \cap B) \cup (A' \cap C')$ | 20. $(C' \cap A) \cup (C' \cap B')$ |
| 21. $(A \cup B \cup C)'$            | 22. $(A \cap B \cap C)'$            |
| 23. $(A \cup B)' \cap C$            | 24. $(B \cup C)' \cap A$            |

In Exercises 25–32, use the Venn diagram shown to answer each question.



25. Which regions represent set  $B$ ?
26. Which regions represent set  $C$ ?
27. Which regions represent  $A \cup C$ ?
28. Which regions represent  $B \cup C$ ?
29. Which regions represent  $A \cap B$ ?
30. Which regions represent  $A \cap C$ ?
31. Which regions represent  $B'$ ?
32. Which regions represent  $C'$ ?

In Exercises 33–44, use the Venn diagram to represent each set in roster form.

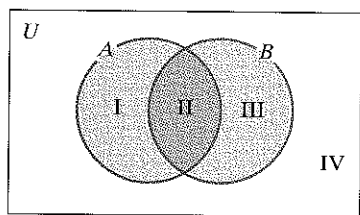


- |                       |                          |                          |
|-----------------------|--------------------------|--------------------------|
| 33. $A$               | 34. $B$                  | 35. $A \cup B$           |
| 36. $B \cup C$        | 37. $(A \cup B)'$        | 38. $(B \cup C)'$        |
| 39. $A \cap B$        | 40. $A \cap C$           | 41. $A \cap B \cap C$    |
| 42. $A \cup B \cup C$ | 43. $(A \cap B \cap C)'$ | 44. $(A \cup B \cup C)'$ |

In Exercises 45–48, construct a Venn diagram illustrating the given sets.

45.  $A = \{4, 5, 6, 8\}$ ,  $B = \{1, 2, 4, 5, 6, 7\}$ ,  
 $C = \{3, 4, 7\}$ ,  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
46.  $A = \{a, e, h, i\}$ ,  $B = \{b, c, e, f, h, i\}$ ,  
 $C = \{e, f, g\}$ ,  $U = \{a, b, c, d, e, f, g, h, i\}$
47.  $A = \{+, -, \times, \div, \rightarrow, \leftrightarrow\}$   
 $B = \{\times, \div, \rightarrow\}$   
 $C = \{\wedge, \vee, \rightarrow, \leftrightarrow\}$   
 $U = \{+, -, \times, \div, \wedge, \vee, \rightarrow, \leftrightarrow, \sim\}$
48.  $A = \{x_3, x_9\}$   
 $B = \{x_1, x_2, x_3, x_5, x_6\}$   
 $C = \{x_3, x_4, x_5, x_6, x_9\}$   
 $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$

Use the Venn diagram shown to solve Exercises 49–52.



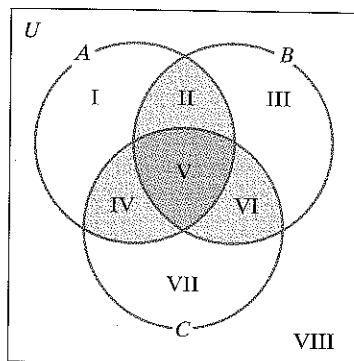
49. a. Which region is represented by  $A \cap B$ ?  
 b. Which region is represented by  $B \cap A$ ?  
 c. Based on parts (a) and (b), what can you conclude?
50. a. Which regions are represented by  $A \cup B$ ?  
 b. Which regions are represented by  $B \cup A$ ?  
 c. Based on parts (a) and (b), what can you conclude?
51. a. Which region(s) is/are represented by  $(A \cap B)'$ ?  
 b. Which region(s) is/are represented by  $A' \cap B'$ ?  
 c. Based on parts (a) and (b), are  $(A \cap B)'$  and  $A' \cap B'$  equal for all sets  $A$  and  $B$ ? Explain your answer.

52. a. Which region(s) is/are represented by  $(A \cup B)'$ ?  
 b. Which region(s) is/are represented by  $A' \cup B'$ ?  
 c. Based on parts (a) and (b), are  $(A \cup B)'$  and  $A' \cup B'$  equal for all sets  $A$  and  $B$ ? Explain your answer.

In Exercises 53–58, use the Venn diagram for Exercises 49–52 in the previous column to determine whether the given sets are equal for all sets  $A$  and  $B$ .

- |                                   |                                 |
|-----------------------------------|---------------------------------|
| 53. $A' \cup B$ , $A \cap B'$     | 54. $A' \cap B$ , $A \cup B'$   |
| 55. $(A \cup B)'$ , $(A \cap B)'$ | 56. $(A \cup B)'$ , $A' \cap B$ |
| 57. $(A' \cap B)'$ , $A \cup B'$  | 58. $(A \cup B)'$ , $A' \cap B$ |

Use the Venn diagram shown to solve Exercises 59–62.



59. a. Which regions are represented by  $(A \cap B) \cup C$ ?  
 b. Which regions are represented by  $(A \cup C) \cap (B \cup C)$ ?  
 c. Based on parts (a) and (b), what can you conclude?
60. a. Which regions are represented by  $(A \cup B) \cap C$ ?  
 b. Which regions are represented by  $(A \cap C) \cup (B \cap C)$ ?  
 c. Based on parts (a) and (b), what can you conclude?
61. a. Which regions are represented by  $A \cap (B \cup C)$ ?  
 b. Which regions are represented by  $A \cup (B \cap C)$ ?  
 c. Based on parts (a) and (b), are  $A \cap (B \cup C)$  and  $A \cup (B \cap C)$  equal for all sets  $A$ ,  $B$ , and  $C$ ? Explain your answer.
62. a. Which regions are represented by  $C \cup (B \cap A)$ ?  
 b. Which regions are represented by  $C \cap (B \cup A)$ ?  
 c. Based on parts (a) and (b), are  $C \cup (B \cap A)$  and  $C \cap (B \cup A)$  equal for all sets  $A$ ,  $B$ , and  $C$ ? Explain your answer.

In Exercises 63–68, use the Venn diagram shown above to determine which statements are true for all sets  $A$ ,  $B$ , and  $C$ , and, consequently, are theorems.

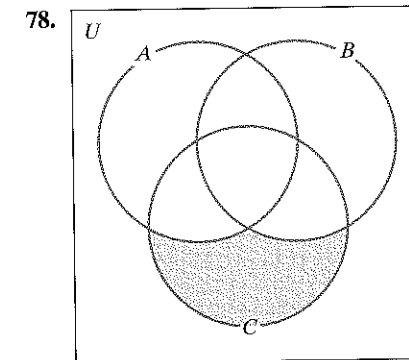
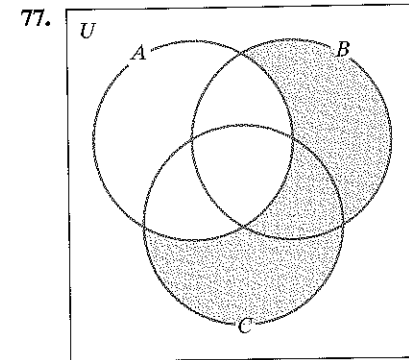
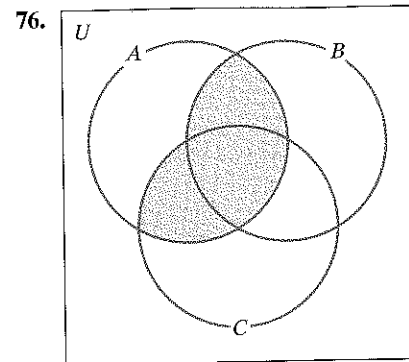
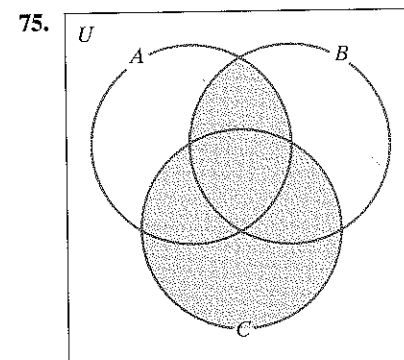
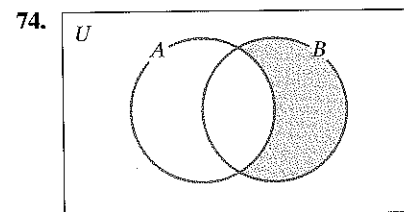
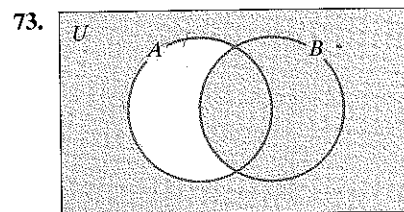
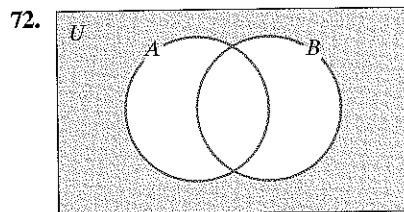
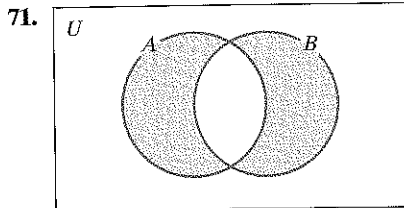
63.  $A \cap (B \cup C) = (A \cap B) \cup C$   
 64.  $A \cup (B \cap C) = (A \cup B) \cap C$   
 65.  $B \cup (A \cap C) = (A \cup B) \cap (B \cup C)$   
 66.  $B \cap (A \cup C) = (A \cap B) \cup (B \cap C)$   
 67.  $A \cap (B \cup C)' = A \cap (B' \cap C')$   
 68.  $A \cup (B \cap C)' = A \cup (B' \cup C')$

### • Practice Plus

69. a. Let  $A = \{c\}$ ,  $B = \{a, b\}$ ,  $C = \{b, d\}$ , and  $U = \{a, b, c, d, e, f\}$ . Find  $A \cup (B' \cap C')$  and  $(A \cup B') \cap (A \cup C')$ .  
 b. Let  $A = \{1, 3, 7, 8\}$ ,  $B = \{2, 3, 6, 7\}$ ,  $C = \{4, 6, 7, 8\}$ , and  $U = \{1, 2, 3, \dots, 8\}$ . Find  $A \cup (B' \cap C')$  and  $(A \cup B') \cap (A \cup C')$ .

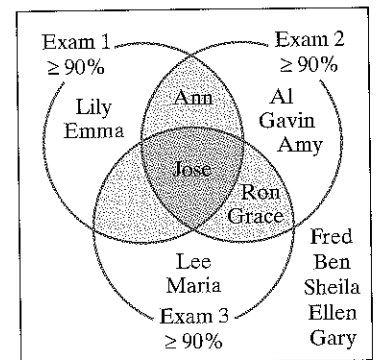
- c. Based on your results in parts (a) and (b), use inductive reasoning to write a conjecture that relates  $A \cup (B' \cap C')$  and  $(A \cup B)' \cap (A \cup C')$ .
  - d. Use deductive reasoning to determine whether your conjecture in part (c) is a theorem.
70. a. Let  $A = \{3\}$ ,  $B = \{1, 2\}$ ,  $C = \{2, 4\}$ , and  $U = \{1, 2, 3, 4, 5, 6\}$ . Find  $(A \cup B)' \cap C$  and  $A' \cap (B' \cap C)$ .
- b. Let  $A = \{d, f, g, h\}$ ,  $B = \{a, c, f, h\}$ ,  $C = \{c, e, g, h\}$ , and  $U = \{a, b, c, \dots, h\}$ . Find  $(A \cup B)' \cap C$  and  $A' \cap (B' \cap C)$ .
- c. Based on your results in parts (a) and (b), use inductive reasoning to write a conjecture that relates  $(A \cup B)' \cap C$  and  $A' \cap (B' \cap C)$ .
  - d. Use deductive reasoning to determine whether your conjecture in part (c) is a theorem.

In Exercises 71–78, use the symbols  $A, B, C, \cap, \cup,$  and  $'$ , as necessary, to describe each shaded region. More than one correct symbolic description may be possible.



• Application Exercises

A math tutor working with a small study group has classified students in the group by whether or not they scored 90% or above on each of three tests. The results are shown in the Venn diagram



In Exercises 79–90, use the Venn diagram to represent each set in roster form.

- 79. The set of students who scored 90% or above on exam 1.
- 80. The set of students who scored 90% or above on exam 2.

81. The set of students who scored 90% or above on exam 1 and exam 3
82. The set of students who scored 90% or above on exam 1 and exam 2
83. The set of students who scored 90% or above on exam 1 and not on exam 2
84. The set of students who scored 90% or above on exam 3 and not on exam 1
85. The set of students who scored 90% or above on exam 1 or not on exam 2
86. The set of students who scored 90% or above on exam 3 or not on exam 1
87. The set of students who scored 90% or above on *exactly one* test
88. The set of students who scored 90% or above on *at least two* tests
89. The set of students who scored 90% or above on exam 2 and not on exam 1 and exam 3
90. The set of students who scored 90% or above on exam 1 and not on exam 2 and exam 3
91. Use the Venn diagram shown at the bottom of the previous page to describe a set of students that is the empty set.
92. Use the Venn diagram shown at the bottom of the previous page to describe the set {Fred, Ben, Sheila, Ellen, Gary}.

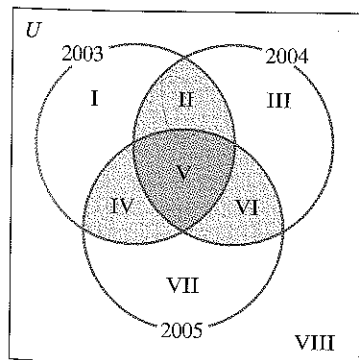
The chart shows the most popular shows on television in 2003, 2004, and 2005.

**MOST POPULAR TELEVISION SHOWS**

| 2003                              | 2004                           | 2005                           |
|-----------------------------------|--------------------------------|--------------------------------|
| 1. <i>Friends</i>                 | 1. <i>Friends</i>              | 1. <i>CSI</i>                  |
| 2. <i>CSI</i>                     | 2. <i>CSI</i>                  | 2. <i>American Idol</i>        |
| 3. <i>E.R.</i>                    | 3. <i>American Idol</i>        | 3. <i>Desperate Housewives</i> |
| 4. <i>Everybody Loves Raymond</i> | 4. <i>Apprentice</i>           | 4. <i>Without a Trace</i>      |
| 5. <i>Law and Order</i>           | 5. <i>Desperate Housewives</i> | 5. <i>Survivor</i>             |
| 6. <i>Survivor</i>                | 6. <i>E.R.</i>                 | 6. <i>Grey's Anatomy</i>       |

Source: Nielsen Media Research

In Exercises 93–98, use the Venn diagram to indicate in which region, I through VIII, each television show should be placed.



93. *Friends*
94. *Desperate Housewives*
95. *CSI*
96. *60 Minutes*
97. *Everybody Loves Raymond*
98. *Grey's Anatomy*

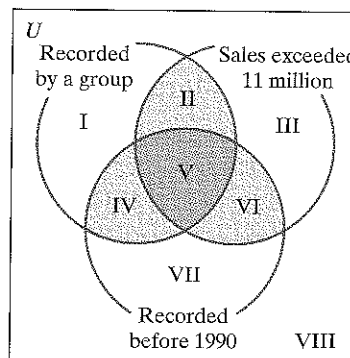
The chart shows the top single recordings of all time.

**TOP SINGLE RECORDINGS**

| Title                      | Artist or Group           | Sales      | Year Released |
|----------------------------|---------------------------|------------|---------------|
| "Candle in the Wind"       | Elton John                | 37 million | 1997          |
| "White Christmas"          | Bing Crosby               | 30 million | 1942          |
| "Rock Around the Clock"    | Bill Haley and His Comets | 17 million | 1954          |
| "I Want to Hold Your Hand" | The Beatles               | 12 million | 1963          |
| "It's Now or Never"        | Elvis Presley             | 10 million | 1960          |
| "Hey Jude"                 | The Beatles               | 10 million | 1968          |
| "I Will Always Love You"   | Whitney Houston           | 10 million | 1992          |
| "Hound Dog"                | Elvis Presley             | 9 million  | 1956          |
| "Diana"                    | Paul Anka                 | 9 million  | 1957          |
| "I'm a Believer"           | The Monkees               | 8 million  | 1966          |

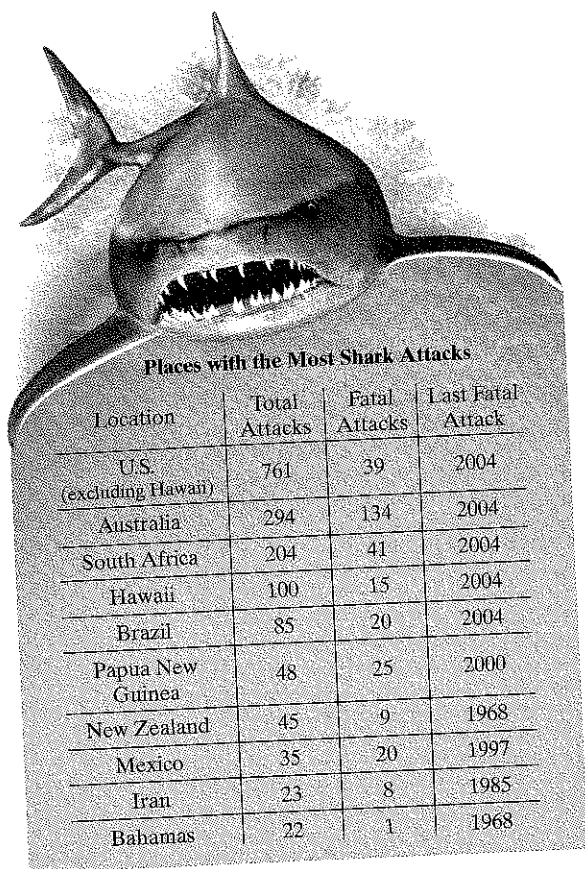
Source: RIAA

In Exercises 99–104, use the Venn diagram to indicate in which region, I through VIII, each recording should be placed.



99. "Candle in the Wind"
100. "White Christmas"
101. "I Want to Hold Your Hand"
102. "Hey Jude"
103. "Diana"
104. "I'm a Believer"

105. The chart shows the ten places where the greatest number of people have been attacked by sharks.



Source: International Shark Attack File

Let  $U$  = the set of locations shown in the chart,  $A$  = the set of locations with at least 100 total attacks,  $B$  = the set of locations with no more than 40 fatal attacks, and  $C$  = the set of locations where the last fatal attack occurred in 2004. Use the information in the chart to construct a Venn diagram that illustrates these sets.

106. The chart shows the ten films nominated for the most Oscars.

**FILMS WITH THE MOST OSCAR NOMINATIONS**

| Film   | Nominations | Awards | Year |
|--|-------------|--------|------|
| <i>All About Eve</i>                                     | 14          | 6      | 1950 |
| <i>Titanic</i>   | 14          | 11     | 1997 |
| <i>Gone with the Wind</i>                                | 13          | 8      | 1939 |
| <i>From Here to Eternity</i>                             | 13          | 8      | 1953 |
| <i>Shakespeare in Love</i>                               | 13          | 7      | 1998 |
| <i>Mary Poppins</i>                                      | 13          | 5      | 1964 |
| <i>Who's Afraid of Virginia Woolf?</i>                   | 13          | 5      | 1966 |
| <i>Forrest Gump</i>                                      | 13          | 6      | 1994 |
| <i>The Lord of the Rings: The Fellowship of the Ring</i> | 13          | 4      | 2001 |
| <i>Chicago</i>   | 13          | 6      | 2004 |

Source: Academy of Motion Picture Arts and Sciences

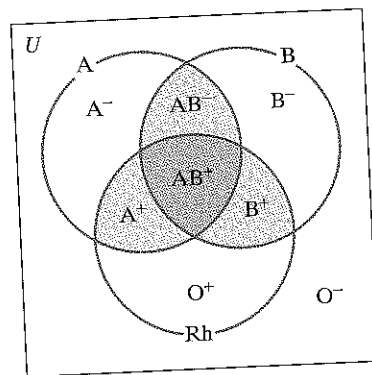
Using abbreviated film titles, let  $U = \{Eve, Titanic, Wind, Eternity, Love, Poppins, Woolf, Gump, Ring, Chicago\}$ ,  $A$  = the set of films nominated for 14 Oscars,  $B$  = the set of films that won at least 7 Oscars, and  $C$  = the set of films that won Oscars after 1965. Use the information in the chart to construct a Venn diagram that illustrates these sets.

• **Writing in Mathematics**

107. If you are given four sets,  $A$ ,  $B$ ,  $C$ , and  $U$ , describe what is involved in determining  $(A \cup B)' \cap C$ . Be as specific as possible in your description.
108. Describe how a Venn diagram can be used to prove that  $(A \cup B)'$  and  $A' \cap B'$  are equal sets.

• **Critical Thinking Exercises**

The eight blood types discussed in the Blitzer Bonus on page 86 are shown once again in the Venn diagram. In blood transfusions, the set of antigens in a donor's blood must be a subset of the set of antigens in a recipient's blood. Thus, the recipient must have all or more of the antigens present in the donor's blood. Use this information to solve Exercises 109–112.



Human blood types

109. What is the blood type of a universal recipient?
110. What is the blood type of a universal donor?
111. Can an  $A^+$  person donate blood to an  $A^-$  person?
112. Can an  $A^-$  person donate blood to an  $A^+$  person?

• **Group Exercises**

113. Each group member should find out his or her blood type. (If you cannot obtain this information, select a blood type that you find appealing!) Read the introduction to Exercises 109–112. Referring to the Venn diagram for these exercises, each group member should determine all other group members to whom blood can be donated and from whom it can be received.
114. The group should define three sets, each of which categorizes  $U$ , the set of students in the group, in different ways. Examples include the set of students with blonde hair, the set of students no more than 23 years old, and the set of students whose major is undecided. Once you have defined the sets, construct a Venn diagram with three intersecting sets and eight regions. Each student should determine which region he or she belongs to. Illustrate the sets by writing each first name in the appropriate region.