

- b. The manufacturer is bound by the following constraints:
- Equipment in the factory allows for making at most 450 console televisions in one month.
  - Equipment in the factory allows for making at most 200 wide-screen televisions in one month.
  - The cost to the manufacturer per unit is \$600 for the console televisions and \$900 for the wide-screen televisions. Total monthly costs cannot exceed \$360,000.

Write a system of three inequalities that describes these constraints.

- c. Graph the system of inequalities in part (b). Use only the first quadrant and its boundary, because  $x$  and  $y$  must both be nonnegative.
- d. Evaluate the objective function for total monthly profit at each of the five vertices of the graphed region. [The vertices should occur at  $(0, 0)$ ,  $(0, 200)$ ,  $(300, 200)$ ,  $(450, 100)$ , and  $(450, 0)$ .]
- e. Complete the missing portions of this statement: The television manufacturer will make the greatest profit by manufacturing \_\_\_\_\_ console televisions each month and \_\_\_\_\_ wide-screen televisions each month. The maximum monthly profit is \$\_\_\_\_\_.

Use the two steps for solving a linear programming problem, given in the box on page 416, to solve the problems in Exercises 15–18.

15. Food and clothing are shipped to victims of a natural disaster. Each carton of food will feed 12 people, while each carton of clothing will help 5 people. Each 20-cubic-foot box of food weighs 50 pounds and each 10-cubic-foot box of clothing weighs 20 pounds. The commercial carriers transporting food and clothing are bound by the following constraints:

- The total weight per carrier cannot exceed 19,000 pounds.
- The total volume must be no more than 8000 cubic feet.

How many cartons of food and clothing should be sent with each plane shipment to maximize the number of people who can be helped?

16. You are about to take a test that contains computation problems worth 6 points each and word problems worth 10 points each. You can do a computation problem in 2 minutes and a word problem in 4 minutes. You have 40 minutes to take the test and may answer no more than 12 problems. Assuming you answer all the problems attempted correctly, how many of each type of problem must you answer to maximize your score? What is the maximum score?
17. A theater is presenting a program on drinking and driving for students and their parents. The proceeds will be donated to a local alcohol information center. Admission is \$2.00 for parents and \$1.00 for students. However, the situation has two constraints: The theater can hold no more than 150 people and every two parents must bring at least one student. How many parents and students should attend to raise the maximum amount of money?

18. On June 24, 1948, the former Soviet Union blocked all land and water routes through East Germany to Berlin. A gigantic airlift was organized using American and British planes to bring food, clothing, and other supplies to the more than 2 million people in West Berlin. The cargo capacity was 30,000 cubic feet for an American plane and 20,000 cubic feet for a British plane. To break the Soviet blockade, the Western Allies had to maximize cargo capacity, but were subject to the following restrictions:

- No more than 44 planes could be used.
- The larger American planes required 16 personnel per flight, double that of the requirement for the British planes. The total number of personnel available could not exceed 512.
- The cost of an American flight was \$9000 and the cost of a British flight was \$5000. Total weekly costs could not exceed \$300,000.

Find the number of American and British planes that were used to maximize cargo capacity.

### • Writing in Mathematics

19. What kinds of problems are solved using the linear programming method?
20. What is an objective function in a linear programming problem?
21. What is a constraint in a linear programming problem? How is a constraint represented?
22. In your own words, describe how to solve a linear programming problem.
23. Describe a situation in your life in which you would like to maximize something, but you are limited by at least two constraints. Can linear programming be used in this situation? Explain your answer.

### • Critical Thinking Exercise

24. Suppose that you inherit \$10,000. The will states how you must invest the money. Some (or all) of the money must be invested in stocks and bonds. The requirements are that at least \$3000 be invested in bonds, with expected returns of \$0.08 per dollar, and at least \$2000 be invested in stocks, with expected returns of \$0.12 per dollar. Because the stocks are medium risk, the final stipulation requires that the investment in bonds should never be less than the investment in stocks. How should the money be invested so as to maximize your expected returns?

### • Group Exercises

25. Group members should choose a particular field of interest. Research how linear programming is used to solve problems in that field. If possible, investigate the solution of a specific practical problem. Present a report on your findings, including the contributions of George Dantzig, Narendra Karmarkar, and L. G. Khachian to linear programming.
26. Members of the group should interview a business executive who is in charge of deciding the product mix for a business. How are production policy decisions made? Are other methods used in conjunction with linear programming? What are these methods? What sort of academic background, particularly in mathematics, does this executive have? Present a group report addressing these questions, emphasizing the role of linear programming for the business.