

EXAMPLE 6 USING THE PYTHAGOREAN THEOREM

- a. A wheelchair ramp with a length of 122 inches has a horizontal distance of 120 inches. What is the ramp's vertical distance?
- b. Construction laws are very specific when it comes to access ramps for the disabled. Every vertical rise of 1 inch requires a horizontal run of 12 inches. Does this ramp satisfy the requirement?

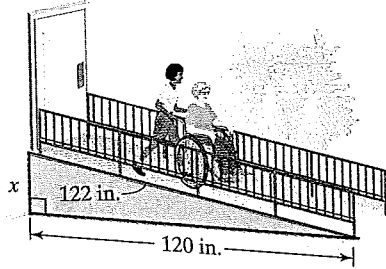


FIGURE 10.23

SOLUTION

- a. Figure 10.23 shows the right triangle that is formed by the ramp, the wall, and the ground. We can find x , the ramp's vertical distance, using the Pythagorean Theorem.

$$\begin{array}{ccccccc} (\text{leg})^2 & \text{plus} & (\text{leg})^2 & \text{equals} & (\text{hypotenuse})^2 \\ x^2 & + & 120^2 & = & 122^2 \end{array}$$

$$x^2 + 120^2 = 122^2$$

This is the equation resulting from the Pythagorean Theorem.

$$x^2 + 14,400 = 14,884$$

Square 120 and 122.

$$x^2 = 484$$

Isolate x^2 by subtracting 14,400 from both sides.

$$x = \sqrt{484} = 22$$

Solve for x by taking the positive square root of 484.

The ramp's vertical distance is 22 inches.

- b. Every vertical rise of 1 inch requires a horizontal run of 12 inches. Because the ramp has a vertical distance of 22 inches, it requires a horizontal distance of $22(12)$ inches, or 264 inches. The horizontal distance is only 120 inches, so this ramp does not satisfy construction laws for access ramps for the disabled.



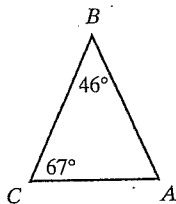
A radio tower is supported by two wires that are each 130 yards long and attached to the ground 50 yards from the base of the tower. How tall is the tower?

EXERCISE SET 10.2

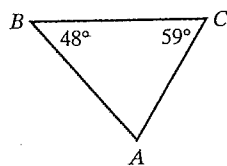
• Practice Exercises

In Exercises 1–4, find the measure of angle A for the triangle shown.

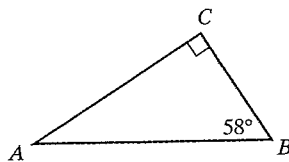
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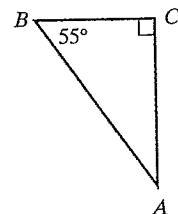
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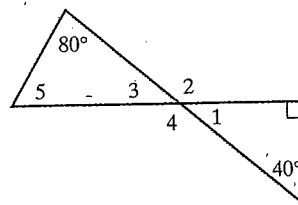


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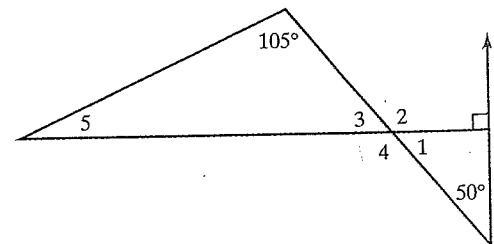


In Exercises 5–6, find the measures of angles 1 through 5 in the figure shown.

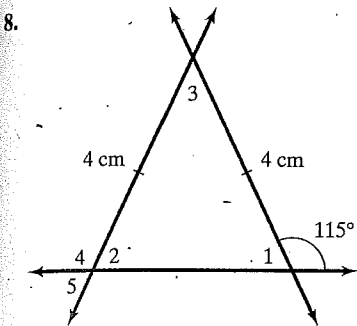
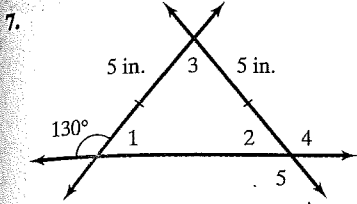
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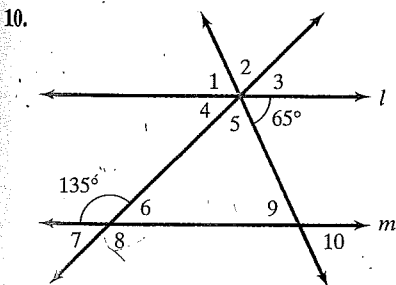
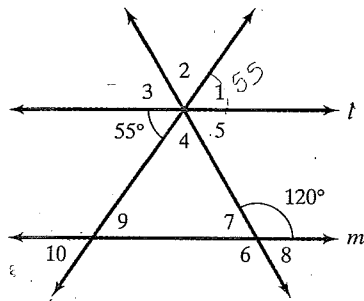
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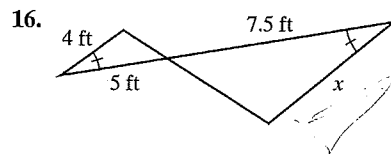
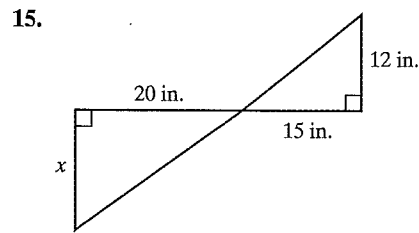
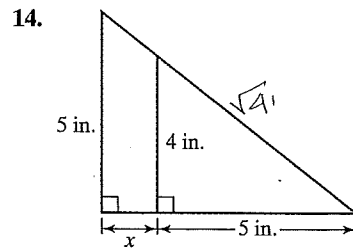
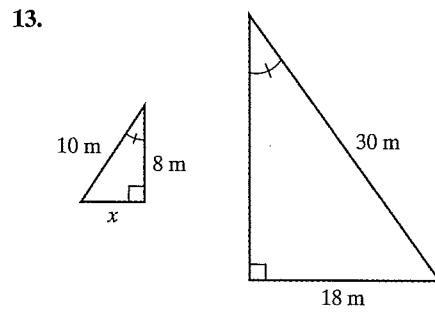
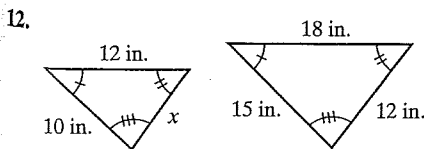
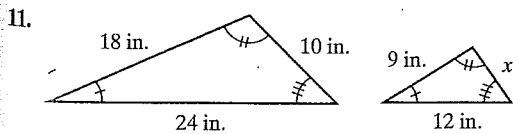
We have seen that isosceles triangles have two sides of equal length. The angles opposite these sides have the same measure. In Exercises 7–8, use this information to help find the measure of each numbered angle.



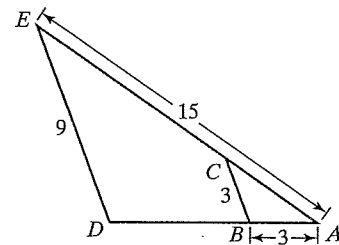
In Exercises 9–10, lines l and m are parallel. Find the measure of each numbered angle.



In Exercises 11–16, use similar triangles and the fact that corresponding sides are proportional to find the missing length, x .



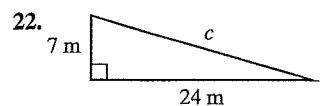
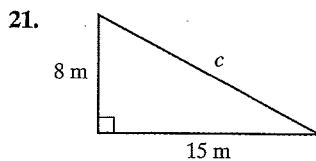
In Exercises 17–19, $\triangle ABC$ and $\triangle ADE$ are similar. Find the length of the indicated side.

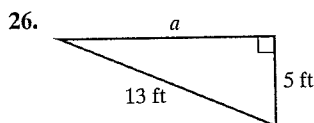
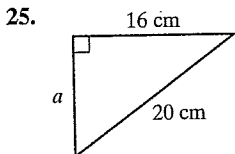
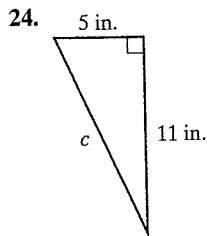
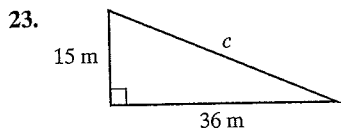


17. \overline{CA} 18. \overline{DB} 19. \overline{DA}

20. In the diagram for Exercises 17–19, suppose that you are not told that $\triangle ABC$ and $\triangle ADE$ are similar. Instead, you are given that \overline{ED} and \overline{CB} are parallel. Under these conditions, explain why the triangles must be similar.

In Exercises 21–26, use the Pythagorean Theorem to find the missing length in each right triangle. Use your calculator to find square roots, rounding, if necessary, to the nearest tenth.

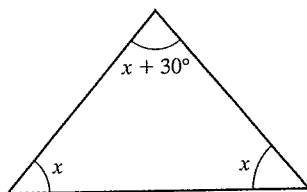




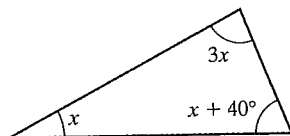
• Practice Plus

Use algebraic equations to solve Exercises 27–30.

27. Two angles of a triangle have the same measure and the third angle is 30° greater than the measure of the other two. Find the measure of each angle.



28. One angle of a triangle is three times as large as another. The measure of the third angle is 40° more than that of the smallest angle. Find the measure of each angle.



29. One angle of a triangle is twice as large as another. The measure of the third angle is 20° more than that of the smallest angle. Find the measure of each angle.
30. One angle of a triangle is three times as large as another. The measure of the third angle is 30° greater than that of the smallest angle. Find the measure of each angle.
31. The Pythagorean Theorem tells us that if a triangle is a right triangle, then $a^2 + b^2 = c^2$. State the converse of the Pythagorean Theorem.

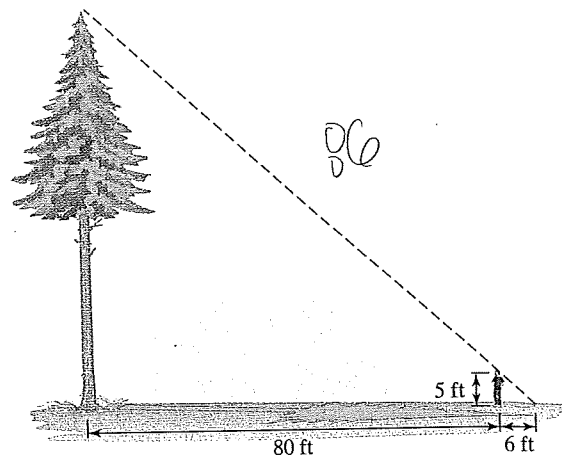
In Exercises 32–34, use the fact that the converse of the Pythagorean Theorem is a true statement to determine if each triangle with sides of the given lengths is a right triangle.

32. 10 cm, 24 cm, 26 cm
 33. 4 m, 8 m, 9 m
 34. $\sqrt{2}$ ft, $\sqrt{7}$ ft, 3 ft

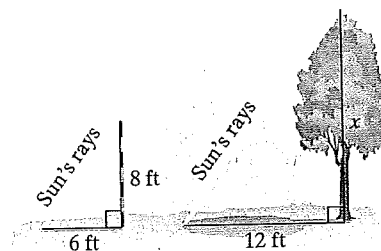
• Application Exercises

Use similar triangles to solve Exercises 35–36.

35. A person who is 5 feet tall is standing 80 feet from the base of a tree, and the tree casts an 86-foot shadow. The person's shadow is 6 feet in length. What is the tree's height?

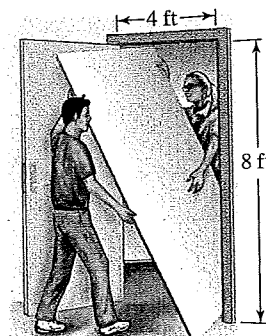


36. A tree casts a shadow 12 feet long. At the same time, a vertical rod 8 feet high casts a shadow that is 6 feet long. How tall is the tree?

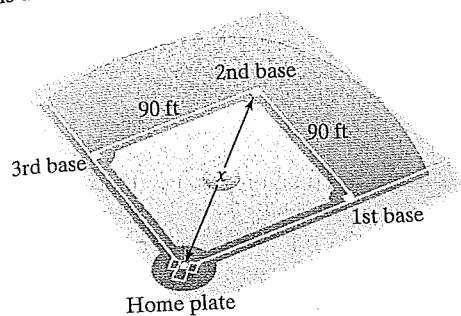


Use the Pythagorean Theorem to solve Exercises 37–45. Use your calculator to find square roots, rounding, if necessary, to the nearest tenth.

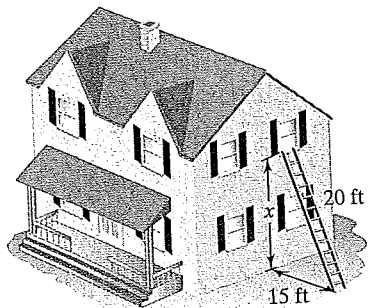
37. The doorway into a room is 4 feet wide and 8 feet high. What is the length of the longest rectangular panel that can be taken through this doorway diagonally?



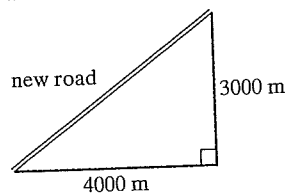
38. A baseball diamond is actually a square with 90-foot sides. What is the distance from home plate to second base?



39. A 20-foot ladder is 15 feet from the house. How far up the house does the ladder reach?

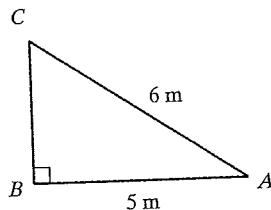


40. A supporting wire is to be attached to the top of a 50-foot antenna. If the wire must be anchored 50 feet from the base of the antenna, what length of wire is required?
41. A supporting wire is to be attached to the top of a 70-foot antenna. If the wire must be anchored 70 feet from the base of the antenna, what length of wire is required?
42. A rectangular park is 6 miles long and 3 miles wide. How long is a pedestrian route that runs diagonally across the park?
43. A rectangular park is 4 miles long and 2 miles wide. How long is a pedestrian route that runs diagonally across the park?
44. If construction costs are \$150,000 per kilometer, find the cost of building the new road in the figure shown.



45. **Picky, Picky, Picky** This problem appeared on Arizona's high school exit exam:

Alex is building a ramp for a bike competition. He has two rectangular boards. One board is six meters and the other is five meters long. If the ramp has to form a right triangle, what should its height be?



Students were asked to select the correct answer from the following options:

3 meters; 4 meters; 3.3 meters; 7.8 meters.

- Among the available choices, which option best expresses the ramp's height? How many feet, to the nearest tenth of a foot, is this? Does a bike competition that requires riders to jump off these heights seem realistic? (ouch!)
- Express the ramp's height to the nearest hundredth of a meter. By how many centimeters does this differ from the "correct" answer on the test? How many inches, to the nearest half inch, is this? Is it likely that a carpenter with a tape measure would make this error?
- According to the problem, Alex has boards that measure 5 meters and 6 meters. A 6-meter board? How many feet, to the nearest tenth of a foot, is this? When was the last time you found a board of this length at Home Depot? (Source: *The New York Times*, April 24, 2005)

• Writing in Mathematics

- If the measures of two angles of a triangle are known, explain how to find the measure of the third angle.
- Can a triangle contain two right angles? Explain your answer.
- What general assumption did Euclid make about a point and a line in order to prove that the sum of the measures of the angles of a triangle is 180° ?
- What are similar triangles?
- If the ratio of the corresponding sides of two similar triangles is 1 to 1 ($\frac{1}{1}$), what must be true about the triangles?
- What are corresponding angles in similar triangles?
- Describe how to identify the corresponding sides in similar triangles.
- In your own words, state the Pythagorean Theorem.
- In the 1939 movie *The Wizard of Oz*, upon being presented with a Th.D. (Doctor of Thinkology), the Scarecrow proudly exclaims, "The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side." Did the Scarecrow get the Pythagorean Theorem right? In particular, describe four errors in the Scarecrow's statement.

