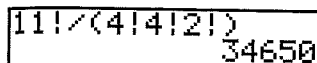


TECHNOLOGY

Parentheses are necessary to enclose the factorials in the denominator when using a calculator to find

$$\frac{11!}{4!4!2!}$$



EXAMPLE 6

USING THE FORMULA FOR PERMUTATIONS OF DUPLICATE ITEMS

In how many distinct ways can the letters of the word MISSISSIPPI be arranged?

SOLUTION The word contains 11 letters ($n = 11$), where four Is are identical ($p = 4$), four Ss are identical ($q = 4$), and 2 Ps are identical ($r = 2$). The number of distinct permutations is

$$\frac{n!}{p!q!r!} = \frac{11!}{4!4!2!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 34,650$$

There are 34,650 distinct ways the letters in the word MISSISSIPPI can be arranged.



In how many ways can the letters of the word OSMOSIS be arranged?

EXERCISE SET 11.2

• Practice and Application Exercises

Use the Fundamental Counting Principle to solve Exercises 1–12.

- Six performers are to present their comedy acts on a weekend evening at a comedy club. How many different ways are there to schedule their appearances?
- Five singers are to perform on a weekend evening at a night club. How many different ways are there to schedule their appearances?
- In the *Cambridge Encyclopedia of Language* (Cambridge University Press, 1987), author David Crystal presents five sentences that make a reasonable paragraph regardless of their order. The sentences are as follows:
 - ✓ Mark had told him about the foxes.
 - ✓ John looked out of the window.
 - ✓ Could it be a fox?
 - ✓ However, nobody had seen one for months.
 - ✓ He thought he saw a shape in the bushes.

In how many different orders can the five sentences be arranged?

- In how many different ways can a police department arrange eight suspects in a police lineup if each lineup contains all eight people?
- As in Exercise 1, six performers are to present their comedy acts on a weekend evening at a comedy club. One of the performers insists on being the last stand-up comic of the evening. If this performer's request is granted, how many different ways are there to schedule the appearances?
- As in Exercise 2, five singers are to perform at a night club. One of the singers insists on being the last performer of the evening. If this singer's request is granted, how many different ways are there to schedule the appearances?
- You need to arrange nine of your favorite books along a small shelf. How many different ways can you arrange the books, assuming that the order of the books makes a difference to you?
- You need to arrange ten of your favorite photographs on the mantel above a fireplace. How many ways can you arrange the photographs, assuming that the order of the pictures makes a difference to you?

In Exercises 9–10, use the five sentences that are given in Exercise 3.

- How many different five-sentence paragraphs can be formed if the paragraph begins with “He thought he saw a shape in the bushes” and ends with “John looked out of the window”?
- How many different five-sentence paragraphs can be formed if the paragraph begins with “He thought he saw a shape in the bushes” followed by “Mark had told him about the foxes”?
- A television programmer is arranging the order that five movies will be seen between the hours of 6 P.M. and 4 A.M. Two of the movies have a G rating, and they are to be shown in the first two time blocks. One of the movies is rated NC-17, and it is to be shown in the last of the time blocks, from 2 A.M. until 4 A.M. Given these restrictions, in how many ways can the five movies be arranged during the indicated time blocks?
- A camp counselor and six campers are to be seated along a picnic bench. In how many ways can this be done if the counselor must be seated in the middle and a camper who has a tendency to engage in food fights must sit to the counselor's immediate left?

In Exercises 13–32, evaluate each factorial expression.

- | | | |
|----------------------------------|-----------------------------|----------------------------------|
| 13. $\frac{9!}{6!}$ | 14. $\frac{12!}{10!}$ | 15. $\frac{29!}{25!}$ |
| 16. $\frac{31!}{28!}$ | 17. $\frac{19!}{11!}$ | 18. $\frac{17!}{9!}$ |
| 19. $\frac{600!}{599!}$ | 20. $\frac{700!}{699!}$ | 21. $\frac{104!}{102!}$ |
| 22. $\frac{106!}{104!}$ | 23. $7! - 3!$ | 24. $6! - 3!$ |
| 25. $(7 - 3)!$ | 26. $(6 - 3)!$ | 27. $\left(\frac{12}{4}\right)!$ |
| 28. $\left(\frac{45}{9}\right)!$ | 29. $\frac{7!}{(7 - 2)!}$ | 30. $\frac{8!}{(8 - 5)!}$ |
| 31. $\frac{13!}{(13 - 3)!}$ | 32. $\frac{17!}{(17 - 3)!}$ | |

In Exercises 33–40, use the formula for ${}_nP_r$ to evaluate each expression.

33. ${}_9P_4$ 34. ${}_7P_3$ 35. ${}_8P_5$
 36. ${}_{10}P_4$ 37. ${}_6P_6$ 38. ${}_9P_9$
 39. ${}_8P_0$ 40. ${}_6P_0$

Use the formula for ${}_nP_r$ to solve Exercises 41–48.

41. A club with ten members is to choose three officers—president, vice-president, and secretary-treasurer. If each office is to be held by one person and no person can hold more than one office, in how many ways can those offices be filled?
42. A corporation has seven members on its board of directors. In how many different ways can it elect a president, vice-president, secretary, and treasurer?
43. For a segment of a radio show, a disc jockey can play 7 records. If there are 13 records to select from, in how many ways can the program for this segment be arranged?
44. Suppose you are asked to list, in order of preference, the three best movies you have seen this year. If you saw 20 movies during the year, in how many ways can the three best be chosen and ranked?
45. In a race in which six automobiles are entered and there are no ties, in how many ways can the first three finishers come in?
46. In a production of *West Side Story*, eight actors are considered for the male roles of Tony, Riff, and Bernardo. In how many ways can the director cast the male roles?
47. Nine bands have volunteered to perform at a benefit concert, but there is only enough time for five of the bands to play. How many lineups are possible?
48. How many arrangements can be made using four of the letters of the word COMBINE if no letter is to be used more than once?

Use the formula for the number of permutations of duplicate items to solve Exercises 49–56.

49. In how many distinct ways can the letters of the word DALLAS be arranged?
50. In how many distinct ways can the letters of the word SCIENCE be arranged?
51. How many distinct permutations can be formed using the letters of the word TALLAHASSEE?
52. How many distinct permutations can be formed using the letters of the word TENNESSEE?

53. In how many ways can the digits in the number 5,446,60 be arranged?
54. In how many ways can the digits in the number 5,432,4 be arranged?

In Exercises 55–56, a signal can be formed by running different colored flags up a pole, one above the other.

55. Find the number of different signals consisting of eight flags that can be made using three white flags, four red flags, and one blue flag.
56. Find the number of different signals consisting of nine flags that can be made using three white flags, five red flags, and one blue flag.

• Writing in Mathematics

57. What is a permutation?
58. Explain how to find $n!$, where n is a positive integer.
59. Explain the best way to evaluate $\frac{900!}{899!}$ without a calculator.
60. Describe what ${}_nP_r$ represents.
61. Write a word problem that can be solved by evaluating ${}_5P_3$.
62. Write a word problem that can be solved by evaluating ${}_7P_3$.
63. If 24 permutations can be formed using the letters in the word BAKE, why can't 24 permutations also be formed using the letters in the word BABE? How is the number of permutations in BABE determined?

• Critical Thinking Exercises

64. Ten people board an airplane that has 12 aisle seats. In how many ways can they be seated if they all select aisle seats?
65. Six horses are entered in a race. If two horses are tied for first place, and there are no ties among the other four horses, in how many ways can the six horses cross the finish line?
66. Performing at a concert are eight rock bands and eight jazz groups. How many ways can the program be arranged if the first, third, and eighth performers are jazz groups?
67. Five men and five women line up at a checkout counter in a store. In how many ways can they line up if the first person in line is a woman, and the people in line alternate woman, man, woman, man, and so on?
68. How many four-digit odd numbers less than 6000 can be formed using the digits 2, 4, 6, 7, 8, and 9?
69. Express ${}_nP_{n-2}$ without using factorials.

SECTION 11.3 • COMBINATIONS

OBJECTIVES

1. Distinguish between permutation and combination problems.
2. Solve problems involving combinations using the combinations formula.

As the twentieth century drew to a close, *Time* magazine presented a series of special issues on the most influential people of the century. In its issue on heroes and icons (June 14, 1999), *Time* discussed a number of people whose careers became more profitable after their tragic deaths, including Marilyn Monroe, James Dean, Jim Morrison, Kurt Cobain, and Selena.

Imagine that you ask your friends the following question: "Of these five people, which three would you select to be included in a documentary featuring the best of their work?" You are not asking your friends to rank their three favorite artists in any kind of order—they should merely select the three to be included in the documentary.