Next, we find the number of ways of selecting 2 Democrats out of 46 Democrats without regard to order. We are looking for the number of combinations of $n = 46$ people taken $r = 2$ people at a time. Once again, we use the formula

$$ {}_nC_r = \frac{n!}{(n-r)!\,r!}. $$

This time, $n = 46$ and $r = 2$.

$$ {}_{46}C_2 = \frac{46!}{(46-2)!\,2!} = \frac{46!}{44!\,2!} = \frac{46 \cdot 45 \cdot \cancel{44!}}{\cancel{44!} \cdot 2 \cdot 1} = 1035 $$

There are 1035 choices for forming 2-member Democratic committees.

We use the Fundamental Counting Principle to find the number of committees that can be formed:

$$ {}_{54}C_3 \cdot {}_{46}C_2 = 24{,}804 \cdot 1035 = 25{,}672{,}140. $$

Thus, 25,672,140 committees can be formed.

○

CHECK POINT 4 The U.S. Senate of the 107th Congress consisted of 50 Democrats, 49 Republicans, and one Independent. How many committees can be formed if each committee must have 3 Democrats and 2 Republicans?

EXERCISE SET 11.3

• Practice Exercises

In Exercises 1–4, does the problem involve permutations or combinations? Explain your answer. (It is not necessary to solve the problem.)

1. A medical researcher needs 6 people to test the effectiveness of an experimental drug. If 13 people have volunteered for the test, in how many ways can 6 people be selected?
2. Fifty people purchase raffle tickets. Three winning tickets are selected at random. If first prize is \$1000, second prize is \$500, and third prize is \$100, in how many different ways can the prizes be awarded?
3. How many different four-letter passwords can be formed from the letters A, B, C, D, E, F, and G if no repetition of letters is allowed?
4. Fifty people purchase raffle tickets. Three winning tickets are selected at random. If each prize is \$500, in how many different ways can the prizes be awarded?

In Exercises 5–20, use the formula for ${}_nC_r$ to evaluate each expression.

5. ${}_6C_5$
6. ${}_8C_7$
7. ${}_9C_5$
8. ${}_{10}C_6$
9. ${}_{11}C_4$
10. ${}_{12}C_5$
11. ${}_8C_1$
12. ${}_7C_1$
13. ${}_7C_7$
14. ${}_4C_4$
15. ${}_{30}C_3$
16. ${}_{25}C_4$
17. ${}_5C_0$
18. ${}_6C_0$
19. $\dfrac{{}_7C_3}{{}_5C_4}$
20. $\dfrac{{}_{10}C_3}{{}_6C_4}$

• Practice Plus

In Exercises 21–28, evaluate each expression.

21. $\dfrac{{}_7P_3}{3!} - {}_7C_3$
22. $\dfrac{{}_{20}P_2}{2!} - {}_{20}C_2$
23. $1 - \dfrac{{}_3P_2}{{}_4P_3}$
24. $1 - \dfrac{{}_5P_3}{{}_{10}P_4}$
25. $\dfrac{{}_7C_3}{{}_5C_4} - \dfrac{98!}{96!}$
26. $\dfrac{{}_{10}C_3}{{}_6C_4} - \dfrac{46!}{44!}$
27. $\dfrac{{}_4C_2 \cdot {}_6C_1}{{}_{18}C_3}$
28. $\dfrac{{}_5C_1 \cdot {}_7C_2}{{}_{12}C_3}$

• Application Exercises

Use the formula for ${}_nC_r$ to solve Exercises 29–36.

29. An election ballot asks voters to select three city commissioners from a group of six candidates. In how many ways can this be done?
30. A four-person committee is to be elected from an organization's membership of 11 people. How many different committees are possible?
31. Of 12 possible books, you plan to take 4 with you on vacation. How many different collections of 4 books can you take?
32. There are 14 standbys who hope to get seats on a flight, but only 6 seats are available on the plane. How many different ways can the 6 people be selected?
33. You volunteer to help drive children at a charity event to the zoo, but you can fit only 8 of the 17 children present in your van. How many different groups of 8 children can you drive?
34. Of the 100 people in the U.S. Senate, 18 serve on the Foreign Relations Committee. How many ways are there to select Senate members for this committee (assuming party affiliation is not a factor in the selection)?
35. To win at LOTTO in the state of Florida, one must correctly select 6 numbers from a collection of 53 numbers (1 through 53). The order in which the selection is made does not matter. How many different selections are possible?

36. To win in the New York State lottery, one must correctly select 6 numbers from 59 numbers. The order in which the selection is made does not matter. How many different selections are possible?

In Exercises 37–46, solve by the method of your choice.

37. In a race in which six automobiles are entered and there are no ties, in how many ways can the first four finishers come in?
38. A book club offers a choice of 8 books from a list of 40. In how many ways can a member make a selection?
39. A medical researcher needs 6 people to test the effectiveness of an experimental drug. If 13 people have volunteered for the test, in how many ways can 6 people be selected?
40. Fifty people purchase raffle tickets. Three winning tickets are selected at random. If first prize is $1000, second prize is $500, and third prize is $100, in how many different ways can the prizes be awarded?
41. From a club of 20 people, in how many ways can a group of three members be selected to attend a conference?
42. Fifty people purchase raffle tickets. Three winning tickets are selected at random. If each prize is $500, in how many different ways can the prizes be awarded?
43. How many different four-letter passwords can be formed from the letters A, B, C, D, E, F, and G if no repetition of letters is allowed?
44. Nine comedy acts will perform over two evenings. Five of the acts will perform on the first evening. How many ways can the schedule for the first evening be made?
45. Using 15 flavors of ice cream, how many cones with three different flavors can you create if it is important to you which flavor goes on the top, middle, and bottom?
46. Baskin-Robbins offers 31 different flavors of ice cream. One of its items is a bowl consisting of three scoops of ice cream, each a different flavor. How many such bowls are possible?

Use the formula for ${}_nC_r$ and the Fundamental Counting Principle to solve Exercises 47–50.

47. In how many ways can a committee of four men and five women be formed from a group of seven men and seven women?
48. How many different committees can be formed from 5 professors and 15 students if each committee is made up of 2 professors and 10 students?
49. The U.S. Senate of the 109th Congress consisted of 55 Republicans, 44 Democrats, and 1 Independent. How many committees can be formed if each committee must have 4 Republicans and 3 Democrats?
50. A mathematics exam consists of 10 multiple-choice questions and 5 open-ended problems in which all work must be shown. If an examinee must answer 8 of the multiple-choice questions and 3 of the open-ended problems, in how many ways can the questions and problems be chosen?

Thousands of jokes have been told about marriage and divorce. Exercises 51–58 are based on the following observations:

- *"By all means, marry; if you get a good wife, you'll be happy. If you get a bad one, you'll become a philosopher." - Socrates*
- *"My wife and I were happy for 20 years. Then we me - Rodney Dangerfield*
- *"Whatever you may look like, marry a man your o age. As your beauty fades, so will his eyesight." - Phy Diller*
- *"Why do Jewish divorces cost so much? Beca they're worth it." - Henny Youngman*
- *"I think men who have a pierced ear are better prepa for marriage. They've experienced pain and bo jewelry." - Rita Rudner*
- *"For a while we pondered whether to take a vacatic get a divorce. We decided that a trip to Bermuda is in two weeks, but a divorce is something you al have." - Woody Allen*

51. In how many ways can these six jokes be ranked fro to worst?
52. If Socrates's thoughts about marriage are excluded many ways can the remaining five jokes be ranke best to worst?
53. In how many ways can people select their three jokes from these thoughts about marriage and div
54. In how many ways can people select their two jokes from these thoughts about marriage and div
55. If the order in which these jokes are told difference in terms of how they are received, h ways can they be delivered if Socrates's comm scheduled first and Dangerfield's joke is told last
56. If the order in which these jokes are told difference in terms of how they are received, h ways can they be delivered if a joke by a woma or Diller) is told first?
57. In how many ways can people select their fa told by a woman (Rudner or Diller) and their t jokes told by a man?
58. In how many ways can people select their fa told by a woman (Rudner or Diller) and favorite jokes told by a man?

• Writing in Mathematics

59. What is a combination?
60. Explain how to distinguish between perm combination problems.
61. Write a word problem that can be solved b ${}_7C_3$.

• Critical Thinking Exercises

62. Write a word problem that can be solved ${}_{10}C_3 \cdot {}_7C_2$.
63. A 6/53 lottery involves choosing 6 of the 1 through 53 and a 5/36 lottery involves ch numbers from 1 through 36. The order in w bers are chosen does not matter. Which lot win? Explain your answer.
64. If the number of permutations of n obje time is six times the number of combinati taken r at a time, determine the value of r. information to determine the value of n? Y

65. In a group of 20 people, how long will it take each person to shake hands with each of the other persons in the group, assuming that it takes three seconds for each shake and only 2 people can shake hands at a time? What if the group is increased to 40 people?

66. A sample of 4 telephones is selected from a shipment of 20 phones. There are 5 defective telephones in the shipment. How many of the samples of 4 phones do not include any of the defective ones?

SECTION 11.4 ◦ FUNDAMENTALS OF PROBABILITY

OBJECTIVES

1. Compute theoretical probability.
2. Compute empirical probability.

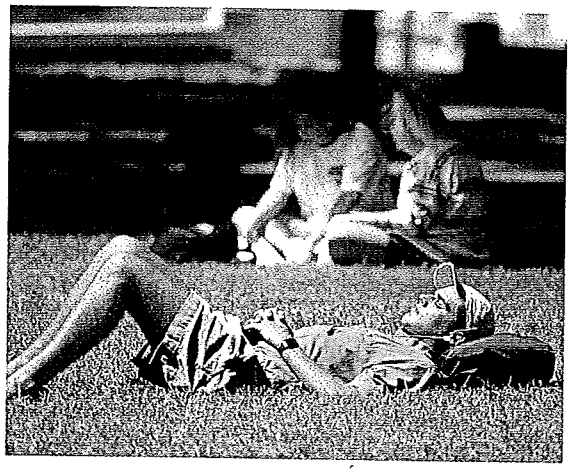
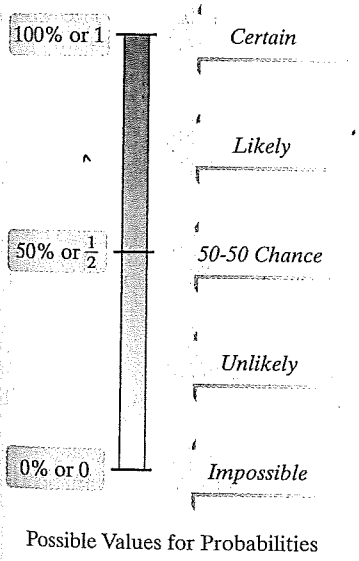


TABLE 11.1 THE HOURS OF SLEEP AMERICANS GET ON A TYPICAL NIGHT

Hours of Sleep	Number of Americans, in millions
4 or less	12
5	27
6	75
7	90
8	81
9	9
10 or more	6

Total: 300

Source: Discovery Health Media



Possible Values for Probabilities

How many hours of sleep do you typically get each night? Table 11.1 indicates that 75 million out of 300 million Americans are getting six hours of sleep on a typical night. The *probability* of an American getting six hours of sleep on a typical night is $\frac{75}{300}$. This fraction can be reduced to $\frac{1}{4}$, or expressed as 0.25, or 25%. Thus, 25% of Americans get six hours of sleep each night.

We find a probability by dividing one number by another. Probabilities are assigned to an *event*, such as getting six hours of sleep on a typical night. Events that are certain to occur are assigned probabilities of 1, or 100%. For example, the probability that a given individual will eventually die is 1. Although Woody Allen whined, "I don't want to achieve immortality through my work. I want to achieve it through not dying," death (and taxes) are always certain. By contrast, if an event cannot occur, its probability is 0. Regrettably, the probability that Elvis will return and serenade us with one final reprise of "Don't Be Cruel" (and we hope we're not) is 0.

Probabilities of events are expressed as numbers ranging from 0 to 1, or 0% to 100%. The closer the probability of a given event is to 1, the more likely it is that the event will occur. The closer the probability of a given event is to 0, the less likely it is that the event will occur.

1 Compute theoretical probability.

Theoretical Probability

You toss a coin. Although it is equally likely to land either heads up, denoted by H, or tails up, denoted by T, the actual outcome is uncertain. Any occurrence for which the outcome is uncertain is called an **experiment**. Thus, tossing a coin is an example of an experiment. The set of all possible outcomes of an experiment is the **sample space** of the experiment, denoted by S. The sample space for the coin-tossing experiment is

$$S = \{H, T\}.$$

Lands heads up Lands tails up

An **event**, denoted by E, is any subset of a sample space. For example, the subset $E = \{T\}$ is the event of landing tails up when a coin is tossed.